64. *On Foliation on Complex Spaces*

By Akihiro SAEKI

Department of Mathematical Sciences, University of Tokyo

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§0. Introduction. In this paper, we discuss foliations on reduced complex spaces. On complex manifolds, foliations are defined in two ways: as coherent subsheaves of the sheaf Θ of germs of holomorphic vector fields and of the sheaf Ω of germs of holomorphic 1-forms, satisfying the "integrability conditions". Foliations defined by vector fields and by 1-forms correspond with each other (cf. [1],[5],[6]). We define foliations on complex spaces in two ways, using vector fields and 1-forms, as a natural extension of the cases on manifolds (Definition 1.0). As the case on a complex manifolds, these two definitions are essentially equivalent with each other (Theorem 1.5). We investigate effects of morphisms of complex spaces on foliations on them. Let $X \to Y$ be a proper modification of reduced complex spaces. Then foliations on X and on Y are correspondent with each other (Theorem 3.3). Thus foliations are bimeromorphically invariant. Details of proofs etc. are written in [4].

§1. Coherent foliations on complex spaces. Let $(X, \mathcal{O}_X)$ be a reduced complex space. We use the following notations:

Ω_X: the sheaf of germs of holomorphic 1-forms on X

Θ_X: the sheaf of germs of holomorphic vector fields on X

sp$X$: the underlying topological space of the complex space X.

By definition, $\Theta_X = \Omega_X^*$: the dual of Ω_X. If X is a closed complex subspace of a domain $D \subset C^m$ defined by a coherent $\mathcal{O}_D$-ideal $\mathcal{I}$, note that $\Omega_X = (\Omega_D / \mathcal{O}_D d\mathcal{I}) \mid_X$.

For a coherent $\mathcal{O}_X$-module $\mathcal{S}$, we set

$$\mathrm{Sing}\mathcal{S} := \{x \in X \mid \mathcal{S}_x \text{ is not } \mathcal{O}_{X,x}\text{-free}\}.$$

If the complex space X is reduced, then $\mathrm{Sing}\,\mathcal{S}$ is a thin analytic set in X. For a coherent $\mathcal{O}_X$-submodule $\mathcal{T}$ of $\mathcal{S}$, we use the notation:

$$S(\mathcal{T}) := \mathrm{Sing}\,\mathcal{S} \cup \mathrm{Sing}(\mathcal{S}/\mathcal{T}).$$

$S(\mathcal{T})$ is an analytic set in X satisfying

$$S(\mathcal{T}) \supset \mathrm{Sing}\,\mathcal{T}.$$

On $X - S(\mathcal{T})$, $\mathcal{T}$ is locally a direct summand of $\mathcal{S}$.

Note that

$$\mathrm{Sing}\,X = \mathrm{Sing}\,\Omega_X$$

holds, where $\mathrm{Sing}\,X$ is the singular locus of the complex space X.

Definition 1.0. We define coherent foliations in two ways.

• Definition a) (by 1-forms).

0) A *coherent foliation* on X is a coherent $\mathcal{O}_X$-submodule F of Ω_X satisfying

(1.1) $$dF_x \subset F_x \wedge \Omega_{X,x}$$