63. *Gamma Factors and Plancherel Measures*

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We explicitly calculate gamma factors of Selberg zeta functions and give a neat formula to the associated Plancherel measures. This report supplements the previous one [7]. The details are described in [8] and will be published elsewhere.

§ 1. Selberg zeta functions. We fix the notation for Selberg zeta functions following mainly Selberg[13], Gangolli [5], Fried [4] ($\kappa = 1$), and Wakayama [15]. Let $M = \Gamma \backslash G/K$ be a compact locally symmetric space of rank one. We denote by $Z_M(s)$ the Selberg zeta function:

$$Z_M(s) = \prod_{p \in \mathrm{Prim}(M)} \prod_{\lambda \geq 0} (1 - N(p)^{-s-\lambda})$$

where $\mathrm{Prim}(M)$ is the set of prime geodesics of M with the norm function $N(p) = \exp(\mathrm{length}(p))$ and λ runs over a certain semi-lattice. We recall the following fact: $Z_M(s)$ has an analytic continuation to all $s \in \boldsymbol{C}$ as a meromorphic function of order $\dim M$ and has the following functional equation

$$Z_M(2\rho_0 - s) = Z_M(s) \exp\left(\mathrm{vol}(M) \int_0^{s-\rho_0} \mu_M(it)\,dt\right).$$

Here, $\rho_0 > 0$ and the Plancherel measure $\mu_M(t)$ calculated by Miatello [12] are given as follows (we use renormalized ρ_0, $\mu_M(t)$ and $\mathrm{vol}(M)$ to simplify the constants):

(0) $G = SO(1, 2n-1)$ ($\Leftrightarrow \dim M$: odd)
$\rho_0 = n - 1$, $\mu_M(it)$: polynomial

(1) $G = SO(1, 2n)$, $\rho_0 = n - 1/2$, $\dim M = 2n$,
$\mu_M(it) = (-1)^n P_M(t)\, \pi \tan(\pi t)$,

$$P_M(t) = \frac{2}{(2n-1)!}\, t \prod_{k=1}^{n-1} \left(t^2 - \left(k - \frac{1}{2}\right)^2\right)$$

(2) $G = SU(1, 2n-1)$, $\rho_0 = n - 1/2$, $\dim M = 4n - 2$,
$\mu_M(it) = -P_M(t)\, \pi \tan(\pi t)$,

$$P_M(t) = \frac{2}{(2n-1)!(2n-2)!}\, t \prod_{k=1}^{n-1} \left(t^2 - \left(k - \frac{1}{2}\right)^2\right)^2$$

(3) $G = SU(1, 2n)$, $\rho_0 = n$, $\dim M = 4n$,
$\mu_M(it) = -P_M(t)\, \pi \cot(\pi t)$,

$$P_M(t) = \frac{2}{(2n)!(2n-1)!}\, t^3 \prod_{k=1}^{n-1} (t^2 - k^2)^2$$

(4) $G = Sp(1, n)$, $\rho_0 = n + 1/2$, $\dim M = 4n$,
$\mu_M(it) = P_M(t)\, \pi \tan(\pi t)$,

$$P_M(t) = \frac{2}{(2n+1)!(2n-1)!}\, t\left(t^2 - \left(n - \frac{1}{2}\right)^2\right) \prod_{k=1}^{n-1} \left(t^2 - \left(k - \frac{1}{2}\right)^2\right)^2$$