48. *The Variance of the Single Point Range of Two Dimensional Recurrent Random Walk*

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§ 1. **Introduction and results.** Let $\{X_n\}_{n=1}^{\infty}$ be the sequence of $\boldsymbol{Z}^d$ valued independent identically distributed random variables defined on a probability space $(\Omega, \mathfrak{B}, P)$. Define $S_0 = 0$ and $S_n = \sum_{k=1}^{n} X_k$. The sequence of these random variables $\{S_n\}_{n=0}^{\infty}$ is called a random walk starting at 0. Let Q_n be the number of the distinct lattice points which the random walk visits once and only once in the first n steps. This random variable is called the single point range of the random walk up to time n or merely the single point range.

We assume the random walk is aperiodic, that is, no proper subgroup of the state space contains the set of x such that $P(X_1 = x) > 0$. If there exists a positive integer N_x satisfying that $P(S_n = x) > 0$ whenever $n \geq N_x$ for every $x \in \boldsymbol{Z}^d$, the random walk is called strongly aperiodic.

We obtain the asymptotic behavior of the variance of Q_n and then can show immediately that the weak law of large number is obeyed.

Theorem A. *For a strongly aperiodic, two dimensional random walk with $EX_1 = 0$ and $E|X_1|^2 < \infty$, there exists a positive constant K such that*

$$\lim_{n\to\infty} \frac{(\log n)^6 Var\, Q_n}{n^2} = c_1^4 K,$$

where $c_1 = 2\pi(\det \Sigma)^{\frac{1}{2}}$ and Σ is the covariance matrix of X_1.

Theorem B. *Under the same condition as in Theorem A, it holds that*

$$\lim_{n\to\infty} P(|Q_n - EQ_n| > \varepsilon EQ_n) = 0$$

for any $\varepsilon > 0$.

By the following theorems, we aware that the estimate of $Var\, Q_n$ is different from that in the case $d \geq 3$. Let $p = P(S_n \neq 0\ n = 1, 2, \cdots)$.

Theorem 1 (Hamana [2]). *If $d \geq 4$ and $p < 1$, then there exists a positive constant σ^2 such that*

$$\lim_{n\to\infty} \frac{Var\, Q_n}{n} = \sigma^2.$$

Theorem 2 (Hamana). *If $d = 3$ and $p < 1$, then there exists a slowly varying function $\psi(n)$ such that*

$$\lim_{n\to\infty} \frac{Var\, Q_n}{n\psi(n)} = 1.$$

Let R_n be the number of the distinct sites visited at least once by a random walk. In the two dimensional case, the asymptotic behavior of R_n was derived by Jain and Pruitt. However, this is not similar to $Var\, Q_n$.