42. Product Formula for Twisted MacPherson Classes

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Introduction. The topological Euler characteristic χ is multiplicative, i.e., $\chi(X \times Y) = \chi(X)\chi(Y)$. For a manifold X, a generalization of $\chi(X)$ to higher dimensional cohomology classes is the Chern cohomology class $c^*(X)$, which satisfies the cross-product formula $c^*(X \times Y) = c^*(X) \times c^*(Y)$. For a (possibly singular) compact complex algebraic variety X, a generalization of $\chi(X)$ to higher dimensional homology classes is the Schwartz-MacPherson homology class $c_*(X)$, which in the smooth case is just the Poincaré dual of the usual Chern cohomology class $c^*(X)$ and the 0-th component of which is equal to $\chi(X)$[1, 4, 5]. Very recently, in connection with lifting Schwartz-MacPherson classes to intersection homology [2], the first author [3] proved the product formula for Schwartz-MacPherson classes, i.e., $c_*(X \times Y) = c_*(X) \times c_*(Y)$. The second author [6, 7] defined the "twisted" MacPherson class $c_{t*}(X)$, which includes Schwartz-MacPherson class $c_*(X)$ as a special case, i.e., $c_{1*}(X) = c_*(X)$. The 0-th component of $c_{t*}(X)$ is the "stratified weighted" Euler characteristic $\chi^t(X)$, which is a degree-dim$X$ polynomial of t, involves Euler characteristic of sigularities also and equals to $\chi(X)$ when $t = 1$. In [7] the second author showed the multiplicativity of χ^t, i.e., $\chi^t(X \times Y) = \chi^t(X)\chi^t(Y)$. In this note, by strengthening and modifying the proof of [3] we show the product formula $c_{t*}(X \times Y) = c_{t*}(X) \times c_{t*}(Y)$, thus the product formulae $c_*(X \times Y) = c_*(X) \times c_*(Y)$ and $\chi^t(X \times Y) = \chi^t(X)\chi^t(Y)$ follow as special cases. More generally we show a product formula for the transformation c_{t*} acting on constructible functions with polynomial coefficients (Theorem 4).

1. Preliminaries. The varieties we consider are all (possibly singular) compact complex algebraic varieties. Let $\boldsymbol{F}$ be the constructible function covariant functor, where $\boldsymbol{F}(X)$ is the abelian group freely generated by characteristic functions $\mathbf{1}_W$ for subvarieties W of X. For a morphism $f : X \to Y$ the pushforward $f_* : \boldsymbol{F}(X) \to \boldsymbol{F}(Y)$ is defined by $(f_*\mathbf{1}_W)(y) := \chi(f^{-1}(y) \cap W)$. Let $H_*(; \boldsymbol{Z})$ be the usual $\boldsymbol{Z}$-homology covariant functor. Deligne and Grothendieck conjectured and MacPherson [3] proved that *there exists a unique natural transformation* $c_* : \boldsymbol{F} \to H_*(; \boldsymbol{Z})$ *satisfying the extra condition that* $c_*(\mathbf{1}_X) = c^*(X) \cap [X]$ *for any smooth* X. To construct the transformation c_*, MacPherson first observes that $\boldsymbol{F}(X)$ is also freely

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