40. *Analytic Zariski Decomposition*

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1. Introduction. Let X be a projective variety and let D be a Cartier divisor on X. The following problem is fundamental in algebraic geometry.

Problem 1. *Study the linear system* $|\nu D|$ *for* $\nu \geq 1$.

To this problem, there is a rather well developed theory in the case of $\dim X = 1$. In the case of $\dim X = 2$, in early 60-th, O. Zariski reduced this problem to the case that D is nef(= numerically semipositive) by using his famous Zariski decomposition ([4]).

Recently Fujita, Kawamata etc. generalized the concept of Zariski decompositions to the case of $\dim X \geq 3$ ([1, 2]). The definition is as follows.

Definition 1. *Let X be a projective variety and let D be an $\boldsymbol{R}$-Cartier divisor on X. The expression*

$$D = P + N \,(P,\, N \in Div(X) \otimes \boldsymbol{R})$$

is called a Zariski decomposition of D, if the following conditions are satisfied.

1. *P is nef,*
2. *N is effective,*
3. $H^0(X, \mathcal{O}_X([\nu P])) \simeq H^0(X, \mathcal{O}_X([\nu D]))$ *holds for all* $\nu \in \boldsymbol{Z}_{\geq 0}$ *where* [ ]*'s denote the integral parts of the divisors.*

Although many useful applications of this decomposition have been known ([1, 2, 3]), as for the existence, very little has been known. There is the following (rather optimistic) conjecture.

Conjecture 1. *Let X be a normal projective variety and let D be a pseudoeffective $\boldsymbol{R}$-Cartier divisor on X. Then there exists a modification $f : Y \to X$ such that $f^* D$ admits a Zariski decomposition.*

In this paper, I would like to announce a "weak solution" to this conjecture. Details will be published elsewhere. In this paper, all algebraic varieties are defined over $\boldsymbol{C}$.

2. Statement of the results. Definition 2. *Let X be a normal projective variety and let D be a $\boldsymbol{R}$-Cartier divisor on X. D is called big if*

$$\kappa(D) := \limsup_{\nu \to +\infty} \frac{\log \dim H^0(X, \mathcal{O}_X([\nu D]))}{\log \nu} = \dim X$$

holds. D is called pseudoeffective, if for any ample divisor H, $D + \varepsilon H$ is big for every $\varepsilon > 0$.

Definition 3. *Let M be a complex manifold of dimension n and let $A_c^{p,q}(M)$ denote the space of $C^\infty(p, q)$ forms of compact support on M with usual Fréchet space structure. The dual space $D^{p,q}(M) := A_c^{n-p,n-q}(M)^*$ is called the space of (p, q)-currents on M. The linear operators $\partial : D^{p,q}(M) \to D^{p+1,q}(M)$ and $\bar{\partial}: D^{p,q}(M) \to D^{p,q+1}(M)$ is defined by*