38. Singular Variation of Non-linear Eigenvalues

By Shin OZAWA
Department of Mathematics, Faculty of Sciences, Tokyo Institute of Technology

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1. Introducrion. Recently several papers have appeared concerning semi-linear elliptic boundary value problems. See, for example, Dancer [1], Lin [3], Wang [7] and the literatures cited there.

We consider the following problem. Let M be a bounded domain in $\boldsymbol{R}^3$ with smooth boundary ∂M. Let w be a fixed point in M. Removing an open ball $B(\varepsilon; w)$ of radius ε with the center w from M, we get $M_\varepsilon = M\backslash\overline{B(\varepsilon; w)}$. We consider the minimizing problem $(1.1)_\varepsilon$ for $\varepsilon>0$. Fix $p>1$. We put

$$(1.1)_\varepsilon \qquad \lambda(\varepsilon)=\inf_{X_\varepsilon}\int_{M_\varepsilon}|\nabla u|^2 dx,$$

where $X_\varepsilon=\{u\in H_0^1(M_\varepsilon),\ \|u\|_{L^{p+1}(M_\varepsilon)}=1\}$. We consider the asymptotic behaviour of $\lambda(\varepsilon)$ as ε tends to 0. It is well known that there exists at least one positive solution u_ε which attains $(1.1)_\varepsilon$ in case of $p\in(1,5)$. We know that the minimizer satisfies $-\Delta u_\varepsilon=\lambda(\varepsilon)u_\varepsilon^p$ in M_ε and $u_\varepsilon=0$ on ∂M_ε. we put

$$\lambda=\inf_X\int_M|\nabla u|^2 dx,$$

where $X=\{u\in H_0^1(M),\ \|u\|_{L^{p+1}(M)}=1\}$.

We have the following

Theorem. *Assume that the positive solution of* $-\Delta \boldsymbol{u}=\lambda \boldsymbol{u}^p$ *in* M *under the Dirichlet condition on* ∂M *is unique. Assume also that the ground state solution* u_ε *for* $(1.1)_\varepsilon$ *is unique for any small* $0<\varepsilon\ll 1$. *We assume that* $\mathrm{Ker}\,(\Delta+\lambda(\varepsilon)pu_\varepsilon^{p-1})=\{0\}$ *for* $0<\varepsilon\ll 1$. *Here* u_ε *is the positive minimizer of* $(1.1)_\varepsilon$. *Then,*

$$(1.2) \qquad \lambda(\varepsilon)-\lambda=4\pi\varepsilon u(w)^2+o(\varepsilon)$$

holds for $p\in(1,2)$. *Here* u *is the minimizer with respect to* λ.

Remarks. We do not treat the case $p=1$ here. In fact, if $p=1$, then $\lambda(\varepsilon)$, (λ, respectively) is the first eigenvalue of $-\Delta$ in M_ε (M, respectively) under the Dirichlet condition and we have an analogous result of (1.2). See [6]. The author wanted to generalize the asymptotic formula for $p=1$ to other cases. This is a motivation of our research.

The domain M such that the number of positive solution of $-\Delta \boldsymbol{u}=\lambda \boldsymbol{u}^p$ in M under the Dirichlet condition on ∂M is exactly one is given by Dancer [1], Gidas-Ni-Nirenberg [2].

The author does not know any example of a domain which satisfies the first, the second and the third assumptions in the Theorem. Even if M is a ball with the center w, the author can not prove that the second