36. The Centralizer Algebras of Mixed Tensor Representations of $\mathcal{U}_q(gl_n)$ and the HOMFLY Polynomial of Links[†]

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Introduction. We construct an algebra $H_{N-1,M-1}(a,q)$ with complex parameters a and q. The centralizer algebra of a mixed tensor representation of $\mathcal{U}_q(gl_n)$ is a quotient of it. The HOMFLY polynomial of links in S^3 is equal to a trace of $H_{N-1,M-1}(a,q)$. Each irreducible character of it corresponds to an invariant of links in a solid torus. As an application, we get a formula for the HOMFLY polynomial of satellite links. The detail will be published elsewhere.

1. The centralizer algebra of mixed tensor representation. The quantum group $\mathcal{U}_q(gl_n)$ is the q-analogue of the universal enveloping algebra $\mathcal{U}(gl_n)$. The Lie algebra gl_n acts on $V_n:=\boldsymbol{C}^n$ naturally and it is called the vector representation. This representation can be deformed for the q-analogue $\mathcal{U}_q(gl_n)$ and is also called the *vector* representation. Let V_n^* denote the dual representation of V_n. Since $\mathcal{U}_q(gl_n)$ is a Hopf algebra, it acts on

$$V_n^{(N,M)}:=\underbrace{V_n\otimes\cdots\otimes V_n}_{N\text{ times}}\otimes\underbrace{V_n^*\otimes\cdots\otimes V_n^*}_{M\text{ times}}.$$

This representation is called the *mixed tensor* representation of $\mathcal{U}_q(gl_n)$. Let

$$C_n^{(N,M)}:=\{x\in\operatorname{End}(V_n^{(N,M)})\,|\,xa=ax\text{ for any }a\in\mathcal{U}_q(gl_n)\}.$$

Then $C_n^{(N,M)}$ is an algebra and is called the *centralizer algebra* with respect to $V_n^{(N,M)}$. Jimbo shows in [2] that $C_n^{(N,0)}$ is a quotient of the Iwahori-Hecke algebra $H_{N-1}(q)$. Let q and a be generic complex parameters. In other words, they are not equal to 0 nor any root of unity. Let $H_{N-1,M-1}(a,q)$ be the algebra defined by the following generators and relations.

$$H_{N-1,M-1}(a,q)=\langle T_1^+,\cdots,T_{N-1}^+,T_1^-,\cdots,T_{M-1}^-,E\,|\,T_i^\pm T_{i+1}^\pm T_i^\pm=T_{i+1}^\pm T_i^\pm T_{i+1}^\pm,$$
$$T_i^\pm T_j^\pm=T_j^\pm T_i^\pm\ (|i-j|\geq 2),\quad T_i^\pm T_j^\mp=T_j^\mp T_i^\pm,\quad ET_i^\pm=T_i^\pm E\ (i\geq 2),$$
$$E(T_1^+)^{-1}T_1^-ET_1^+=E(T_1^+)^{-1}T_1^-ET_1^-,\quad ET_1^\pm E=a^{-1}E,\quad E^2=-\frac{a-a^{-1}}{q-q^{-1}}E,$$
$$T_1^+E(T_1^+)^{-1}T_1^-E=T_1^-E(T_1^+)^{-1}T_1^-E,\quad (T_i^\pm-q)(T_i^\pm+q^{-1})=0\rangle.$$

Theorem 1. (1) *The algebra $H_{N-1,M-1}(a,q)$ is semisimple and its dimension is equal to the factorial $(N+M)!$.*

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