33. Notes on Some Classical Series Associated with Discrete Subgroups of $U(1, n; C)$ on $\partial B^n \times \partial B^n \times \partial B^n$

By Shigeyasu KAMIYA
Department of Mechanical Engineering, Okayama University of Science

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Let $U(1, n; C)$ be the group of unitary transformations. In the previous paper [2], we discussed the action of discrete subgroups of $U(1, n; C)$ on $\partial B^n \times \partial B^n \times \cdots \times \partial B^n$, where ∂B^n is the boundary of the complex unit ball. In [4], P. J. Nicholls considered the convergence of some series associated with discrete subgroups of Möbius transformations on the products of the boundary of the unit ball in real n-space.

Our purpose is to show two theorems on some classical series associated with discrete subgroups of $U(1, n; C)$ acting on $\partial B^n \times \partial B^n \times \partial B^n$. Throughout this paper G denotes a discrete subgroup of $U(1, n; C)$. Let $\{g_1, g_2, \cdots\}$ be a complete list of elements of G. If g_k is an element of G, then g_k is represented by a matrix $(a_{ij}^{(k)})_{1 \leq i,j \leq n+1}$. Let $x = (x_1, \cdots, x_n)$, $y = (y_1, \cdots, y_n)$ and $z = (z_1, \cdots, z_n)$ be points in ∂B^n.

Theorem 1. *The series*

$$\sum_{g_k \in G} \left(\left| a_{11}^{(k)} + \sum_{j=2}^{n+1} a_{1j}^{(k)} x_{j-1} \right| \left| a_{11}^{(k)} + \sum_{j=2}^{n+1} a_{1j}^{(k)} y_{j-1} \right| \left| a_{11}^{(k)} + \sum_{j=2}^{n+1} a_{1j}^{(k)} z_{j-1} \right| \right)^{-2n}$$

converges for almost every triple (x, y, z) in $\partial B^n \times \partial B^n \times \partial B^n$.

Theorem 2. *If $\sum_{g_k \in G} |a_{11}^{(k)}|^{-m}$ converges for $m > 0$, then the series*

$$\sum_{g_k \in G} \left(\left| a_{11}^{(k)} + \sum_{j=2}^{n+1} a_{1j}^{(k)} x_{j-1} \right| \left| a_{11}^{(k)} + \sum_{j=2}^{n+1} a_{1j}^{(k)} y_{j-1} \right| \left| a_{11}^{(k)} + \sum_{j=2}^{n+1} a_{1j}^{(k)} z_{j-1} \right| \right)^{-m}$$

converges for every distinct points x, y and z in ∂B^n.

We shall give our proofs.

Proof of Theorem 1. Let $\Gamma(g_k)$ be the set of (x, y, z) in $\partial B^n \times \partial B^n \times \partial B^n$ for which

$$\left| a_{11}^{(k)} + \sum_{j=2}^{n+1} a_{1j}^{(k)} x_{j-1} \right| \left| a_{11}^{(k)} + \sum_{j=2}^{n+1} a_{1j}^{(k)} y_{j-1} \right| \left| a_{11}^{(k)} + \sum_{j=2}^{n+1} a_{1j}^{(k)} z_{j-1} \right| > 1.$$

Set

$$F = \bigcap_{g_k \neq id} \Gamma(g_k).$$

It follows from [2, Theorem 11] that F is a fundamental set for the group action on $\partial B^n \times \partial B^n \times \partial B^n$. Since F is of positive measure and has no G-equivalent points,

$$\sum_{g_k \in G} \sigma^*(g_k(F)) < \infty,$$

where σ^* is the product measure on $\partial B^n \times \partial B^n \times \partial B^n$ derived from the measure σ on ∂B^n (see [2, p. 288]). For $(x, y, z) \in F$

$$\sum_{g_k \in G} \sigma^*(g_k(F)) = \sum_{g_k \in G} \int_{g_k(F)} d\sigma^*$$