31. Retractive Nil-extensions of Regular Semigroups. II

By Stojan BOGDANOVIĆ and Miroslav ĆIRIĆ
Institute of Mathematics, Knez Mihailova 35, Beograd, Yugoslavia

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Abstract: This paper is the continuation of [6]. Here we consider in particular retractive nil-extensions of unions of groups.

By Theorem 1, some criterions for a semigroup to be a retractive nil-extension of a union of groups are given. A characterization of retractive nil-extensions of semilattice of left and right groups (mixed property) is given by Theorem 2. For the related results see [2] and [5].

Throughout this paper, Z^+ will denote the set of all positive integers. A semigroup S is *π-regular*, if for every $a \in S$ there exists $n \in Z^+$ such that $a^n \in a^n S a^n$. Let us denote by $Reg(S)$ ($Gr(S)$, $E(S)$) the set of all regular (completely regular, idempotent) elements of a semigroup S. A semigroup S is *Archimedean*, if for all $a, b \in S$ there exists $n \in Z^+$ such that $a^n \in SbS$. A semigroup S is *completely Archimedean*, if S is Archimedean and has a primitive idempotent (or, equivalently, if it is a nil-extension of a completely simple semigroup [1]). If e is an idempotent of a semigroup S, then by G_e we denote the maximal subgroup of S with e as its identity and $T_e = \{a \in S \mid (\exists n \in Z^+) a^n \in G_e\}$. For undefined notions and notations we refer [1], [10] and [6].

Veronesi's theorem [11]. *A semigroup S is a semilattice of completely Archimedean semigroups, if and only if S is π-regular and $Reg(S) = Gr(S)$.*

Munn's lemma [9]. *Let a be an element of a semigroup S such that a^n lies in some subgroup G of S for some $n \in Z^+$. If e is an identity of G, then $ea = ae \in G_e$ and $a^m \in G_e$ for all $m \in Z^+$, $m \geq n$.*

Lemma 1 [5]. *Let S be a nil-extension of a union of groups K. Then every retraction φ of S onto K has the following representation:*

$$\varphi(x) = xe \qquad \text{if } x \in T_e,\ e \in E(S).$$

Theorem 1. *The following conditions on a semigroup S are equivalent:*

(i) *S is a retractive nil-extension of a union of groups;*

(ii) *S is π-regular and for all $x, a, y \in S$ there exists $n \in Z^+$ such that*

$$(1) \qquad xa^n y \in x^2 S y^2;$$

(iii) *S is a subdirect product of a union of groups and a nil-semigroup.*

Proof. (i)⇒(ii). Let S be a retractive nil-extension of a union of groups K, with the retraction φ of S onto K. Let $x, a, y \in S$. Then there exists $n \in Z^+$ such that $a^n \in K$, so $xa^n y \in K$. Let $x^m \in G_e$, $y^k \in G_f$, $m, k \in Z^+$,

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