3. On the Inseparable Degree of the Gauss Map of Higher Order for Space Curves

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Abstract: Let X be a curve non-degenerate in a projective space $\boldsymbol{P}^N$ defined over an algebraically closed field of positive characteristic p, consider the Gauss map of order m defined by the osculating m-planes at general points of X, and denote by $\{b\}_{0\leq j\leq N}$ the orders of X. We prove that the inseparable degree of the Gauss map of order m is equal to the highest power of p dividing b_{m+1}.

Key words: Space curve, Gauss map, inseparable degree.

0. Introduction. Let X be an irreducible curve in a projective space $\boldsymbol{P}^N$ defined over an algebraically closed field k of positive characteristic p, C the normalization of X, and $\iota: C\to\boldsymbol{P}^N$ the natural morphism. Denote by $\iota^{(m)}: C\to\boldsymbol{G}(\boldsymbol{P}^N, m)$ the Gauss map of order m defined by the osculating m-planes of X, where $\boldsymbol{G}(\boldsymbol{P}^N, m)$ is a Grassmann manifold of m-planes in $\boldsymbol{P}^N$. Assume that X is non-degenerate in $\boldsymbol{P}^N$, and let $\{b_j\}_{0\leq j\leq N}$ be the orders of ι. The purpose of this short note is to prove

Theorem. *The inseparable degree of $\iota^{(m)}$ is the highest power of p dividing b_{m+1}.*

In case of $m=1$, Theorem is known: For $N=2$, see [4, Proposition 4.4]; for a general N, see [5, Remark below Corollary 2.3], [3, Proposition 4]. A corollary to this result will give a generalization of [5, Theorem 2.1] (see Corollary below).

In case of $m=N-1$, Theorem coincides with a result of A. Hefez and N. Kakuta, announced in [1]. Although it has not been published yet, according to Hefez [2], their proof for the theorem is similar in spirit to ours (precisely speaking, of the first version), but not identical. Hefez and Kakuta moreover found

Theorem (Hefez-Kakuta). *Denote by $C^{(m)}X$ the conormal variety of order m, and by $X^{*(m)}$ the m-dual. Then the inseparable degree of the natural morphism $C^{(m)}X\to X^{*(m)}$ is equal to the highest power of p dividing b_{m+1}.*

This result is stated as a theorem in [2] without proof.

We finally mention that this Theorem of Hefez and Kakuta is deduced also from our theorem and a result in [6] that $C^{(m)}X\to X^{*(m)}$ *has the same inseparable degree as* $\iota^{(m)}$, which is proved directly without going through

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