25. *Eisenstein Series on Quaternion Half-space of Degree 2*

By Shoyu NAGAOKA
Department of Mathematics, Kinki University

(Communicated by Shokichi IYANAGA, M. J. A., May 12, 1992)

1. Eisenstein series. Let $\boldsymbol{H}$ denote the skew field of real Hamiltonian quaternions with the canonical basis $e_1=1, e_2, e_3, e_4$. Let $Her(n,\boldsymbol{H})$ denote the real Jordan algebra consisting of all quaternion Hermitian $n\times n$ matrices and $Pos(n,\boldsymbol{H}):=\{Y\in Her(n,\boldsymbol{H})\mid Y>0\}$ the open subset of all positive definite matrices. Then *the quaternion half-space of degree n* is given by

$$\mathcal{H}(n,\boldsymbol{H}):=\{Z=X+iY\mid X\in Her(n,\boldsymbol{H}),\ Y\in Pos(n,\boldsymbol{H})\}\subset Her(n,\boldsymbol{H})\otimes_{\boldsymbol{R}}\boldsymbol{C}.$$

Set $J_n=\begin{pmatrix}0_n & E_n\\ -E_n & 0_n\end{pmatrix}$. The group

$$G_n:=\{M\in M(2n,\boldsymbol{H})\mid {}^t\bar{M}J_nM=qJ_n \text{ for some } q\in\boldsymbol{R}_+\}$$

acts on $\mathcal{H}(n,\boldsymbol{H})$ in the usual way. Given $Z\in\mathcal{H}(n,\boldsymbol{H})$ and $\begin{pmatrix}A & B\\ C & D\end{pmatrix}\in G_n$ with $n\times n$ blocks A, B, C, D set

$$M\langle Z\rangle:=(AZ+B)(CZ+D)^{-1}.$$

The Hurwitz order is denoted

$$\mathcal{O}=\boldsymbol{Z}e_0+\boldsymbol{Z}e_1+\boldsymbol{Z}e_2+\boldsymbol{Z}e_3,\qquad e_0=\frac{1}{2}(e_1+e_2+e_3+e_4)$$

(cf. [1], [4]).

The group

$$\Gamma_n:=\{M\in M(2n,\mathcal{O})\mid {}^t\bar{M}J_nM=J_n\}$$

is called *the modular group of quaternions of degree n*. Let $\Gamma_{n,\infty}$ denote the subgroup of Γ_n defined by

$$\Gamma_{n,\infty}:=\left\{\begin{pmatrix}A & B\\ C & D\end{pmatrix}\in\Gamma_n\mid C=0_n\right\}.$$

Given $A\in M(n,\boldsymbol{H})$, $A^\vee$ denotes the element of $M(2n,\boldsymbol{C})$ obtained by the representation of quaternions as complex 2×2 matrices and we define $\delta(A)=\det^{1/2}(A^\vee)$ (we take as $\delta(A)>0$ for $A\in Pos(n,\boldsymbol{H})$).

We define a kind of Eisenstein series on $\mathcal{H}(n,\boldsymbol{H})$ by

$$E_k^{(n)}(Z,s)=\delta(Y)^{s/2}\sum_{\left(\begin{smallmatrix}* & *\\ C & D\end{smallmatrix}\right)\in\Gamma_{n,\infty}\backslash\Gamma_n}|\delta(CZ+D)|^{-s}\delta(CZ+D)^{-k},$$

where $k\in\boldsymbol{Z}$, $(Z,s)\in\mathcal{H}(n,\boldsymbol{H})\times\boldsymbol{C}$ and $Z=X+iY$. It is known that this series is absolutely convergent if $\mathrm{Re}(s)+k>2(2n-1)$. Put, for $Y\in Pos(n,\boldsymbol{H})$, $H\in Her(n,\boldsymbol{H})$, and $(\alpha,\beta)\in\boldsymbol{C}^2$,

$$\xi^{(n)}(Y,H;\alpha,\beta)=\int_{Her(n,\boldsymbol{H})}\boldsymbol{e}(-\tau(H,V))\delta(V+iY)^{-\alpha}\delta(V-iY)^{-\beta}dV,$$

where τ denotes the reduced trace form, $\boldsymbol{e}(s)=\exp(2\pi is)$ for $s\in\boldsymbol{C}$, and dV is the Euclidean measure on $Her(n,\boldsymbol{H})$ by viewing it as $\boldsymbol{R}^n\times\boldsymbol{H}^{(n(n-1))/2}$ (cf. [8],