24. Spherical Functions on Some p-adic Classical Groups

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Introduction. We write down explicitly the so called Satake transform of Hecke algebra of some $\mathfrak{p}$-adic classical groups (O) with $n\neq 2\nu$, (Sp), (U), (U^+), (U^-), using Macdonald's idea for the $\mathfrak{p}$-adic Chevalley groups ([2]). For our purpose, we have only to evaluate zonal spherical functions (in §2) and the number of double cosets by the maximal compact subgroup K (in §3). The details containing the case (O) with $n=2\nu$ will be published elsewhere.

§1. **Preliminaries.** Let k be a $\mathfrak{p}$-adic field where $\mathfrak{p}$ does not lie over 2. Let k' be either k itself, a quadratic extension of k or the (unique) central division quaternion algebra over k. $\mathcal{O}$ denotes the maximal order of k'. We denote by e the ramification index of k'/k, and $\mathcal{P}=(\Pi)$ (resp. $\mathfrak{p}=(\pi)$) the prime ideal in k' (resp. k). We denote by $x\to\bar{x}$ $(x\in k')$ the canonical involution. Let ε be an element of the center of k' such that $\varepsilon\bar{\varepsilon}=1$, V a right vector space over k' of dimension n, and $\langle\ ,\ \rangle$ a non-degenerate ε-hermitian form on V, i.e., a k-bilinear mapping $V\times V\to k'$ such that

$$\langle x, y\rangle=\varepsilon\overline{\langle y, x\rangle},\quad \langle xa, yb\rangle=\bar{a}\langle x, y\rangle b\quad \text{foy all } x, y\in V,\ a, b\in k'.$$

It is known that we have the following five cases.

(O) $k'=k$ and $\varepsilon=1$.
(Sp) $k'=k$ and $\varepsilon=-1$.
(U) k' is a quadratic extension of k and $\varepsilon=1$.
(U^+) k' is a division quaternion algebra over k and $\varepsilon=1$.
(U^-) k' is a division quaternion algebra over k and $\varepsilon=-1$.

Now, let ν be the Witt index of V and put $n=n_0+2\nu$. There exists a (not uniquely determined) system of vectors $\{e_i, e_i'\ (1\leq i\leq\nu)\}$ such that

$$\langle e_i, e_j\rangle=\langle e_i', e_j'\rangle=0,\quad \langle e_i, e_j'\rangle=\delta_{ij}\quad \text{for all } i, j,$$

(δ_{ij} is Kronecker's symbol). Set

$$V_0=(\Sigma e_i k'+\Sigma e_i' k')^{\perp},\quad L_0=\{x\in V_0\,|\,\langle x, x\rangle\in\mathcal{O}\},\quad L=\Sigma e_i\mathcal{O}+\Sigma e_i'\mathcal{O}+L_0.$$

Then L is a maximal lattice and there is a system of vectors $\{f_i\ (1\leq i\leq n_0)\}$ such that

$$L_0=\Sigma f_i\mathcal{O},\quad \langle f_i, f_j\rangle=0,\quad \text{if } i\neq j.$$

We define α (resp. β) to be the number of $\{f_i\}$ such that $\langle f_i, f_i\rangle\in\mathcal{O}^\times$ (resp. $\langle f_i, f_i\rangle\in\mathcal{P}$). Note that $\alpha+\beta=n_0$.

We now take this basis $\{e_1,\cdots,e_\nu, f_1,\cdots,f_{n_0}, e_\nu',\cdots,e_1'\}$ and identify a k'-linear transformation g of V with a matrix (g_{ij}) by

$$g:(e_1,\cdots,e_1')\to(e_1,\cdots,e_1')(g_{ij}).$$

Let G be the connected component of the group of similitudes of V, that is