22. *Examples of Essentially Non-Banach Representations**)

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Let G be a locally compact unimodular group, K a compact subgroup of G. Let $\{\mathfrak{H}, T(x)\}$ be a topologically irreducible representation of G on a locally convex complete Hausdorff topological vector space $\mathfrak{H}$. We assume there exists an equivalence class δ of irreducible representation of K which is contained finitely many times in $\{\mathfrak{H}, T(x)\}$. Then the subspace $\mathfrak{H}(\delta)$ of all vectors transformed according to δ under $T(k)$, $k \in K$, is finite-dimensional, and there exists a usual projection $E(\delta)$ of $\mathfrak{H}$ onto $\mathfrak{H}(\delta)$. After R. Godement [1] we call the function $\phi_\delta(x) = \mathrm{trace}\,[E(\delta)T(x)]$, $x \in G$, a *spherical function* of type δ. A function $\rho(x)$ on G is called a *seminorm* if it is positive-valued, lower semicontinuous and satisfies $\rho(xy) \leqq \rho(x)\rho(y)$ for $x, y \in G$. If there exists a seminorm $\rho(x)$ such that $|\phi_\delta(x)| \leqq \rho(x)$ for $x \in G$, then ϕ_δ is called *quasi-bounded.* In the case when $\mathfrak{H}$ is a Banach space, the corresponding spherical function ϕ_δ is quasi-bounded.

Even if a spherical function is defined from a non-Banach representation, it can be quasi-bounded, or equivalently equal to the one which is obtained from a Banach representation. For example, in the case when G is a connected semisimple Lie group or a motion group on the plane, all spherical functions are quasi-bounded (cf. [2]). A topologically irreducible representation which defines non-quasi-bounded spherical functions is called an *essentially non-Banach representation.* Here we give examples of essentially non-Banach representations of a semidirect product group $G = S \rtimes K$, where S is a free group with infinitely many generators and K is a compact abelian group.

§1. A semidirect product group $G = S \rtimes K$. We denote by N or Z the set of natural numbers or integers respectively. Let S be a free group with discrete topology generated by infinitely many generators s_n, $n \in N$. The automorphism group $\mathrm{Aut}\langle s_n \rangle$ of the infinite cyclic group $\langle s_n \rangle = \{s_n^m \mid m \in Z\}$ consists of two elements, 1_n the identity and ψ_n the automorphism of $\langle s_n \rangle$ such that $\psi_n(s_n) = s_n^{-1}$. Let $K = \prod_{n \in N} \mathrm{Aut}\langle s_n \rangle$ be the direct product group which is compact with respect to the product topology. Then K is naturally embedded into $\mathrm{Aut}\, S$ as $k \cdot s = k_{n_1}(s_{n_1}^{m_1}) \cdots k_{n_p}(s_{n_p}^{m_p})$ for $k = (k_n) \in K$ and $s = s_{n_1}^{m_1} \cdots s_{n_p}^{m_p} (m_j \in Z, n_j \in N)$. The semidirect product group $G = S \rtimes K$ is locally compact and unimodular. In §2 we will construct K-finite topologically irreducible representations of G which are essentially non-Banach representations.

*) Dedicated to Prof. N. Tatsuuma on his 60th birthday.