14. *The Arithmetic Structure of the Galois Group of the Maximal Nilpotent Extension of an Algebraic Number Field*

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The purpose of this article is to give an exposition of the recent results on the structure of the Galois group of the maximal nilpotent extension of an algebraic number field. Various interesting results have been obtained on the basis of the fact found by Tate (Serre [8]) that the Schur multiplier of the Galois group is trivial.

1. The abelian case. Let k be an algebraic number field of finite degree, and k^{ab} and k^{nil} be its maximal abelian extension and its maximal nilpotent one, respectively, in a fixed algebraic closure $\bar{\boldsymbol{Q}}$ of the rational number field $\boldsymbol{Q}$.

The structure of the Galois group $\mathfrak{A}:=\mathrm{Gal}(k^{\mathrm{ab}}/k)$ is well known by Takagi-Artin class field theory; in particular by Chevalley's idelic formulation of the theory, we can vividly see how the local class field theories on k are tied up as a global whole by the relations determined by the global numbers of k. To be more precise, let us denote the decomposition group of a prime divisor $\mathfrak{p}$ of k in $\mathfrak{A}$ by $\mathfrak{A}_{\mathfrak{p}}$ and the inertia group by $\mathfrak{E}_{\mathfrak{p}}$; then $\mathfrak{A}_{\mathfrak{p}}$ may be identified with a local Galois group $\mathrm{Gal}(k_{\mathfrak{p}}^{\mathrm{ab}}/k_{\mathfrak{p}})$ of the maximal abelian extension $k_{\mathfrak{p}}^{\mathrm{ab}}$ of the completion $k_{\mathfrak{p}}$ of k by $\mathfrak{p}$. Let A be the restricted product of $\mathfrak{A}_{\mathfrak{p}}$ with respect to $\mathfrak{E}_{\mathfrak{p}}$ for all prime divisors of k, and $\alpha: A\rightarrow\mathfrak{A}$ be the continuous homomorphism which is defined by the fixed embedding of $\mathfrak{A}_{\mathfrak{p}}$ into $\mathfrak{A}$ for all $\mathfrak{p}$. Since $\mathfrak{A}$ is generated by Frobenius automorphisms of prime divisors, α is surjective. The local Artin maps of local class field theory also naturally define a continuous homomorphism $a: k_A^{\times}\rightarrow A$ of the idele group $k_A^{\times}$ of k to the restricted product A. The combined homomorphism $\mathfrak{a}:=\alpha\circ a$ is none other than the Artin map of the global class field theory for k; hence we have an exact sequence,

$$1\longrightarrow\overline{a(k^{\times})}\longrightarrow A\xrightarrow{\alpha}\mathfrak{A}\longrightarrow 1, \tag{1.1}$$

where $\overline{a(k^{\times})}$ is the topological closure of the image of $k^{\times}$ by a in A. We see by this how Galois theoretic local-global relations are determined by the global numbers of k.

The main aim of this article is to report the fact that there exists an analogous exact sequence for the Galois group $\mathfrak{G}:=\mathrm{Gal}(k^{\mathrm{nil}}/k)$ which is a natural lifting of the one for $\mathfrak{A}=\mathfrak{G}^{\mathrm{ab}}:=\mathfrak{G}/[\mathfrak{G},\mathfrak{G}]$ (see Section 4). The details and proofs will be found in a forthcoming paper [5] of the author.