# 11. On Automorphism Groups of Compact Riemann Surfaces with Prescribed Group Structure

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Let $X$ be a compact Riemann surface of genus $g\geq 2$ and let $\mathrm{Aut}(X)$ be the group of all conformal automorphisms on $X$. Let $\rho: \mathrm{Aut}(X)\to GL(g, \boldsymbol{C})$ denote the canonical representation for a (fixed) basis $\{\xi_1, \cdots, \xi_g\}$ of abelian differentials of the first kind on $X$. In fact, for a $\sigma\in\mathrm{Aut}(X)$, we define the matrix $(s_{ij})\in GL(g, \boldsymbol{C})$ by the relation:

$$\sigma^*(\xi_i)=\sum_{j=1}^{g} s_{ij}\xi_j \qquad (i=1, \cdots, g).$$

Here $\sigma^*(\xi_i)$ means the pull-back of $\xi_i$ by $\sigma$. We denote by $\rho(AG; X)$ the image of a subgroup $AG$ of $\mathrm{Aut}(X)$ by $\rho$. The purpose of this paper is to investigate conditions for a non abelian subgroup of $GL(g, \boldsymbol{C})$ of order 8 to be conjugate to some $\rho(AG; X)$ (for some $AG$ and some $X$). We say that $G\subset GL(g, \boldsymbol{C})$ arises from a compact Riemann surface of genus $g$ if $G$ has the above property.

A more detailed account will be published elsewhere.

**§1. Preliminaries.** Let $G$ be a finite subgroup of $GL(g, \boldsymbol{C})$ and $H$ a non-trivial cyclic subgroup of $G$. Define two sets $CY(G)$ and $CY(G; H)$ by

$CY(G) := \{K; K \text{ is a non-trivial cyclic subgroup of } G\}$,

$CY(G; H) := \{K\in CY(G); K \text{ contains } H \text{ strictly}\}$.

**Definition** (see [1]). We say that $G$ satisfies the *CY-condition*, if any element of $CY(G)$ is $GL(g, \boldsymbol{C})$-conjugate to a group arising from Riemann surfaces of genus $g$.

**Definition.** We say that $G$ satisfies *E condition* if for every element $A$ of $G$, $\mathrm{Tr}(A)+\mathrm{Tr}(A^{-1})$ is an integer. Further we define as follows:

$r(H) := 2-(\mathrm{Tr}(A)+\mathrm{Tr}(A^{-1}))$, where $H=\langle A\rangle$.

$r_*(H; G)=r(H)-\sum_K r_*(K; G)$, where $K$ ranges over the set $CY(G; H)$.

$l(H; G) := (r_*(H; G))/[N_G(H): H]$ where $N_G(H)$ means the normalizer of $H$ in $G$.

We say that $G$ satisfies the *RH-condition* if $G$ satisfies the $E$ condition and $l(H; G)$ is a non-negative integer for any $H\in CY(G)$.

We denote by $D_8$ and $Q_8$, respectively, the dihedral group of order 8 and quaternion group,

i.e., $D_8=\langle a, b; a^4=b^2=1, b^{-1}ab=a^{-1}\rangle$,
$Q_8=\langle a, b; a^4=1, a^2=b^2, b^{-1}ab=a^{-1}\rangle$.

The character table of $D_8$ is as follows ($Q_8$ has the same character table):