10. *Domains of Square Roots of Regularly Accretive Operators*

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1. Introduction. The purpose of this paper is to give a sufficient condition for the domain of the square root of a regularly accretive operator and that of its adjoint operator to be the same.

Let X and V be two Hilbert spaces with $V \subset X$. Let the inclusion from V into X be continuous, and let V be dense in X. We denote by (f, g) (resp. $(u, v)_V$) the inner product in X (resp. V) and put $\|f\|=(f, f)^{1/2}$ and $\|u\|_V=(u, u)_V^{1/2}$.

Let $a[u, v]$ be a bounded sesquilinear form on $V \times V$;

(1.1) $\qquad |a[u, v]| \leqq M\|u\|_V\|v\|_V, \qquad M>0,$ for any $u, v \in V$.

We suppose that $a[u, v]$ is strongly coercive;

(1.2) $\qquad \operatorname{Re} a[u, u] \geqq \delta\|u\|_V^2, \qquad \delta>0,$ for any $u \in V$.

Let A be the closed operator associated with the variational triple $\{V, X, a\}$, that is, $u \in V$ belongs to $D(A)$ (the domain of A) if and only if there exists $f \in X$ such that $a[u, v]=(f, v)$ for any $v \in V$, and we define $Au=f$. We call A a *regularly accretive operator.*

We define the adjoint form $a^*[u, v]$ by $a^*[u, v]=\overline{a[v, u]}$ for any $u, v \in V$. It is known that the closed operator associated with the variational triple $\{V, X, a^*\}$ is the adjoint operator A^* of A.

As is well known, we can construct the fractional power A^θ $(0 \leqq \theta \leqq 1)$ of the regularly accretive operator A. Kato [3] showed that $D(A^\theta)= D(A^{*\theta}) \subset V$ if $0 \leqq \theta < 1/2$. But generally $D(A^{1/2})=D(A^{*1/2})$ does not hold, for Mcintosh [7] gave a counterexample. On the other hand, Kato and Lions obtained the following results independently.

Theorem A (Kato [4], Lions [6]). *Each of the following condition is sufficient for* $D(A^{1/2})=D(A^{*1/2})=V$.

(i) *Both* $D(A^{1/2})$ *and* $D(A^{*1/2})$ *are oversets* (*or subsets*) *of* V.

(ii) $D(A^\theta)=D(A^{*\theta})$ *for* $\theta=1/2$ *or* 1.

(iii) *There exists a Hilbert space* W *which satisfies* (1) $W \subset X$, (2) V *is a closed subspace of* $[X, W]_{1/2}$, (3) $D(A) \subset W$ *and* $D(A^*) \subset W$, *where* $[X, W]_\theta$ $(0 \leqq \theta \leqq 1)$ *denotes the complex interpolation space of* X *and* W.

Remark 1. Theorem A-(iii) is due only to Lions.

Remark 2. We may replace Theorem A-(ii) with $D(A^\theta)=D(A^{*\theta})$ for some θ with $1/2 \leqq \theta \leqq 1$, because we have $[X, D(A^\theta)]_{1/(2\theta)}=D(A^{1/2})$.

In the next section we give another sufficient condition for $D(A^{1/2})= D(A^{*1/2})=V$.

2. Main result. The sesquilinear form $a[u, v]$ can be written