83. *Embedding into Kac-Moody Algebras and Construction of Folding Subalgebras for Generalized Kac-Moody Algebras*

By Satoshi NAITO
Department of Mathematics, Kyoto University

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Introduction. In the preceding paper [5], we defined a *regular subalgebra* $\bar{\mathfrak{g}}$ of a symmetrizable *Kac-Moody algebra* $\mathfrak{g}(A)$, and showed that $\bar{\mathfrak{g}}$ is isomorphic to a *generalized Kac-Moody algebra* (=GKM algebra) $\mathfrak{g}(\bar{A})$ associated to a canonically defined symmetrizable GGCM $\bar{A}$, as explained below.

In the first half of this paper, we show that a symmetrizable GKM algebra $\mathfrak{g}(A)$ can be embedded into some Kac-Moody algebra as a regular subalgebra under a certain weak condition on the GGCM A. In the latter half of this paper, we introduce and study what we call a *folding subalgebra* of a symmetrizable GKM algebra $\mathfrak{g}(A)$, corresponding to a diagram automorphism π of the GGCM A. This subalgebra is contained in the fixed point subalgebra of an automorphism of $\mathfrak{g}(A)$ induced by π, and is easier to deal with than the fixed point subalgebra itself.

§ 1. Embedding of GKM algebras into Kac-Moody algebras.

1.1. Regular subalgebras. Here, we recall the notion of regular subalgebras of symmetrizable Kac-Moody algebras introduced in [5]. For the detailed accounts, see [2], [5], and [6]. Let $\mathfrak{g}(A)$ be a Kac-Moody algebra associated to a symmetrizable *generalized Cartan matrix* (=GCM) A over the complex number field $\boldsymbol{C}$, and $\mathfrak{h}$ its *Cartan subalgebra.*

Definition 1.1 ([5]). A subset $\bar{\Pi}=\{\beta_r\}_{r=1}^m$ of the root system Δ of $\mathfrak{g}(A)$ is called *fundamental* if it satisfies the following:

(1) $\beta_1, \beta_2, \cdots, \beta_m$ are linearly independent;

(2) $\beta_i-\beta_j \notin \Delta \quad (1\leq i\neq j\leq m)$;

(3) if β_i is an *imaginary root*, then it is a positive root.

For each imaginary root β_i, we define $\beta_i^\vee:=\nu^{-1}(\beta_i)$, where $\nu: \mathfrak{h}\rightarrow\mathfrak{h}^*$ is a linear isomorphism determined by a *standard invariant form* $(\cdot|\cdot)$ on $\mathfrak{g}(A)$. For real root β_i, $\beta_i^\vee$ has been defined as a dual real root of β_i. Then, we proved in [5] that $\bar{A}:=(\bar{a}_{ij})_{i,j=1}^m$ with $\bar{a}_{ij}=\langle\beta_j, \beta_i^\vee\rangle$ is a symmetrizable *generalized GCM* (=GGCM), that is, $\bar{A}$ satisfies the following:

(C1) either $\bar{a}_{ii}=2$ or $\bar{a}_{ii}\leq 0$;

(C2) $\bar{a}_{ij}\leq 0$ if $i\neq j$, and $\bar{a}_{ij}\in \boldsymbol{Z}$ if $\bar{a}_{ii}=2$;

(C3) $\bar{a}_{ij}=0$ implies $\bar{a}_{ji}=0$.

Now, take and fix non-zero root vectors $E_r\in\mathfrak{g}_{\beta_r}$ and $F_r\in\mathfrak{g}_{-\beta_r}$ such that $[E_r, F_r]=\beta_r^\vee$ $(1\leq r\leq m)$. Then,