70. On a Conjecture of Gackstatter and Laine on Some Differential Equations

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1. Introduction. In this paper, we consider the differential equation

$$P(z, w')=Q(z, w), \tag{1.1}$$

in the complex plane, where $P(z, w')$ and $Q(z, w)$ are polynomials of w' and w with meromorphic (maybe transcendental) coefficients, respectively:

$$\begin{cases} P(z, w')=w'^{p}+b_{p-1}(z)w'^{p-1}+\cdots+b_1(z)w' \\ Q(z, w)=a_q(z)w^q+a_{q-1}(z)w^{q-1}+\cdots+a_0(z),\ a_q(z)\not\equiv 0. \end{cases} \tag{1.1'}$$

We use standard notations in Nevanlinna theory [2][5]. Let $f(z)$ be a meromorphic function. As usual, $m(r, f)$, $N(r, f)$ and $T(r, f)$ denote the proximity function, the counting function, and the characteristic function of $f(z)$, respectively.

A function $\varphi(r)$, $0\leq r<\infty$, *is said to be* $S(r, f)$ if there is a set $E\subset \mathbf{R}^+$ of finite linear measure such that $\varphi(r)=o(T(r, f))$ as $r\to\infty$, $r\notin E$. A meromorphic function $a(z)$ is *small with respect to* $f(z)$, if $T(r, a)=S(r, f)$.

Let $\Omega(z, w, w', \cdots, w^{(n)})$ be a differential polynomial of w with meromorphic coefficients and $\mathcal{M}$ be the set of its coefficients. We call a transcendental meromorphic solution $w(z)$ of the differential equation $\Omega(z, w, w', \cdots, w^{(n)})=0$ is *an admissible solution*, if $T(r, a)=S(r, w)$ for any $a(z)\in\mathcal{M}$.

Gackstatter and Laine [1] investigated the binomial equation

$$(w')^p=Q(z, w) \quad (b_{p-1}=\cdots=b_1\equiv 0 \text{ in } (1.1')) \tag{1.2}$$

and they conjectured that it would not possess any admissible solution if $1\leqq q\leqq p-1$. Some investigations have been done for this conjecture, e.g. [6][8][9][10].

In [6], Ozawa pointed out that this conjecture is closely connected with a problem due to Hayman ([3] Problem 1.21). If (1.1) possesses an admissible solution $w(z)$, then from (1.1) and (1.1′).

$$pT(r, w')=qT(r, w)+S(r, w). \tag{1.3}$$

Thus $T(r, w)/T((r, w')\to p/q>1$ for $r\to\infty$ outside a set E of finite linear measure.

Recently, He and Laine [4] solved this conjecture affirmatively.

Theorem A. *When* $1\leqq q\leqq p-1$ *in* (1.2), *the differential equation* (1.2) *possesses no admissible solution.*

Toda [9] treated more general differential equation

$$H(z, w, w', \cdots, w^{(k)})^m=Q(z, w), \tag{1.4}$$

where $H(z, w, w', \cdots, w^{(k)})$ is a differential polynomial of w. He proved the following theorem.