69. *A New One-parameter Family of 2×2 Quantum Matrices*

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(Communicated by Shokichi IYANAGA, M. J. A., Oct. 14, 1991)

We introduce a new one-parameter family of quadratic braided 2×2 matrix bialgebras $B_q(2)$. We work over the complex numbers C. All proofs of this announcement will be included in [5]. The main results were also announced at the AMS San Fransisco meeting in January 1991.

We start with the following R-matrix. Let q be a complex number.

$$R_q=\left[1-\frac{(q-1)^2}{2}\right]e_{11}\otimes e_{11}+\left[1-\frac{(q+1)^2}{2}\right]e_{22}\otimes e_{22}$$
$$+\frac{(q-1)^2}{2}e_{12}\otimes e_{12}+\frac{(q+1)^2}{2}e_{21}\otimes e_{21}$$
$$+\frac{1-q^2}{2}(e_{11}\otimes e_{22}+e_{22}\otimes e_{11})+\frac{1+q^2}{2}(e_{12}\otimes e_{21}+e_{21}\otimes e_{12})$$

where e_{ij} denote the matrix units. A tedious verification shows that R_q satisfies the Yang-Baxter equation (or the braid condition)

$$(I\otimes R_q)(R_q\otimes I)(I\otimes R_q)=(R_q\otimes I)(I\otimes R_q)(R_q\otimes I).$$

Further we have $(R_q-I)(R_q+q^2I)=0$ and when $q\neq 0$, $q^2\neq -1$, R_q is diagonal with two two-dimensional eigenspaces.

Definition 1. Assume $q\neq 0$, $q^2\neq -1$. Let $B_q(2)$ be the C-algebra defined by generators a, b, c, d and the following relations

(1) $ad=da$, (2) $bc=cb$, (3) $ab-\hat{q}ba=(1-\hat{q})cd$,

(4) $dc+\hat{q}cd=(1+\hat{q})ba$, (5) $ac-\hat{q}ca=-(1+\hat{q})bd$,

(6) $db+\hat{q}bd=-(1-\hat{q})ca$, (7) $a^2+b^2=c^2+d^2$,

(8) $(1+\hat{q})b^2=(\hat{q}-1)c^2$,

where $\hat{q}=\dfrac{q+q^{-1}}{2}$.

The above relations are equivalent to saying that the matrix $X\otimes X$ with $X=\begin{pmatrix} a & b\\ c & d\end{pmatrix}$, commutes with R_q. Hence the algebra $B_q(2)$ has a bialgebra structure with comultiplication

$$\Delta\begin{pmatrix} a & b\\ c & d\end{pmatrix}=\begin{pmatrix} a\otimes 1 & b\otimes 1\\ c\otimes 1 & d\otimes 1\end{pmatrix}\begin{pmatrix} 1\otimes a & 1\otimes b\\ 1\otimes c & 1\otimes d\end{pmatrix}.$$

The bialgebra $B_q(2)$ is braided by [2] or [1].

Proposition 2. *Assume* $q\neq 0$, $q^4\neq 1$. *Let*

$$f=\frac{1}{2}(a+d),\quad g=\frac{1}{2}(a-d),\quad s=\frac{1}{2}(q_-b+q_+c),\quad t=\frac{1}{2}(q_-b-q_+c)$$

where $q_\pm=(\sqrt{q}\pm\sqrt{q}^{-1})^{-1}$.

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