7. *A Note on the Problem of Yokoi*

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(Communicated by Shokichi IYANAGA, M. J. A., Jan. 14, 1991)

Let p be a prime congruent to 1 mod 4 and $\varepsilon_p=(t+u\sqrt{p})/2>1$ be the fundamental unit of $\boldsymbol{Q}(\sqrt{p})$. From Theorem 1 of [1], there exist only a finite number of real quadratic fields $\boldsymbol{Q}(\sqrt{p})$ with class number one for any fixed positive integer u. The problem of enumerating these fields for the cases $u=1$ and $u=2$ was solved by H. K. Kim, M.-G. Leu and T. Ono ([2]).

In this paper, we shall determine all these fields for $1\leqq u\leqq 300$ in proving the following theorem.

Theorem. *With the above notation, there exist at most* 44 *real quadratic fields* $\boldsymbol{Q}(\sqrt{p})$ *with class number one for* $1\leqq u\leqq 300$, *where* p *are those in Table* II *with one possible exception.*

Proof. Let χ_p be the Kronecker character belonging to $\boldsymbol{Q}(\sqrt{p})$ and $L(s,\chi_p)$ be the corresponding L-series. Then by Theorem 2 of [4], for any $y\geqq 12$, we have

$$L(1,\chi_p)>\frac{0.655}{y}p^{-1/y}$$

with one possible exception of p, where $y=\log p$.

Further, from class number formula, for any $e^y\leqq p$ ($y\geqq 12$), we have

$$\begin{aligned} h(p)&=\frac{\sqrt{p}}{2\log\varepsilon_p}L(1,\chi_p)\\ &>\frac{0.655}{y}\,\frac{\sqrt{p}\,p^{-1/y}}{2\log(u\sqrt{p})}=\frac{0.655}{y}\,\frac{p^{(y-2)/2y}}{2\log u+\log p}\\ &\geqq\frac{0.655e^{(y-2)/2}}{y(y+2\log u)}. \end{aligned}$$

Thus $h(p)=1$ implies

$$(1)\qquad 0.655e^{(y-2)/2}\leqq y(y+2\log u).$$

Put for convenience

$$g(x,y)=\frac{0.655e^{(y/2)-1}}{y(y+2x)},\ \text{where } x=\log u.$$

The curve C in Figure 1 represents the graph of $g(x,y)=1$. The inequality (1) means that the point ($\log u$, $\log p$) with $h(p)=1$ should lie in the shadowed domain in this figure. In particular, $1\leqq u\leqq 2$ implies $1\leqq p\leqq e^{14}$ and $5\leqq u\leqq 300$ implies $1\leqq p\leqq e^{15}$.

Now put

$$U=\{2^r\prod p_i^{s_i}\,|\,r=0\text{ or }1,\ p_i\equiv 1\ (\mathrm{mod}\ 4),\ s_i\geqq 0\}.$$

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