# 62. Normal Bases and $\lambda$-invariants of Number Fields

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Let $\boldsymbol{Q}$ be the rational number field, $k$ be a number field, i.e. a finite algebraic extension of $\boldsymbol{Q}$, $S$ be a set of prime ideals of $k$ and $L$ a finite algebraic extension of $k$. We denote by $\mathfrak{O}_L$ the integer ring of $L$ and $v_\mathfrak{p}$ an additive valuation of $L$ with respect to a prime ideal $\mathfrak{p}$ of $L$. We denote by $\mathfrak{O}_L(S)$ the ring of elements $\alpha$ in $L$ with $v_\mathfrak{p}(\alpha)\geqq 0$ for all prime ideals $\mathfrak{p}$ of $L$ such that $\mathfrak{p}\cap k$ does not belong to $S$. Now let $p$ be a fixed odd prime number, $\boldsymbol{Z}_p$ the $p$-adic integer ring and $K$ a $\boldsymbol{Z}_p$-extension of $k$. Then there exists a tower of cyclic extensions of $k$

$$k=K_0\subset K_1\subset\cdots\subset K_n\subset\cdots\subset K$$

such that $K_n$ is an extension of $k$ with the degree $[K_n:k]=p^n$. For the cyclotomic $\boldsymbol{Z}_p$-extension $k_\infty$ of $k$, we write $k_n=(k_\infty)_n$.

Recently, Kersten and Michaliček discussed normal bases of $p$-integer rings of intermediate fields of a $\boldsymbol{Z}_p$-extension of a CM-field and Vandiver's conjecture. Furthermore, Fleckinger and Nguyen Quang Do have discussed normal bases of $p$-integer rings of intermediate fields of a $\boldsymbol{Z}_p$-extension of a number field. In this paper, we investigate normal bases of $S$-integer rings of intermediate fields of a $\boldsymbol{Z}_p$-extension of an imaginary quadratic field and the Iwasawa $\lambda$-invariant.

Now we define as follows:

**Definition** (cf. [4]). *We say, a $\boldsymbol{Z}_p$-extension $K/k$ has a normal $S$-basis, if each $\mathfrak{O}_{K_n}(S)/\mathfrak{O}_k(S)$ has a normal basis. Namely, there exists an element $\alpha_n$ of $\mathfrak{O}_{K_n}(S)$ such that $\{\alpha_n^\sigma\mid\sigma\in G(K_n/k)\}$ is a free $\mathfrak{O}_k(S)$-basis of $\mathfrak{O}_{K_n}(S)$, where $G(K_n/k)$ is the Galois group of $K_n$ over $k$.*

Let $F$ be an imaginary quadratic field, $F_\infty$ the cyclotomic $\boldsymbol{Z}_p$-extension of $F$ and $\zeta_n=\exp(2\pi\sqrt{-1}/p^n)$. We put $k=F(\zeta_1)$ and $\Delta=G(k/F)$. Let $\delta$ be the order of $\Delta$ and $\chi:\Delta\to\boldsymbol{Z}_p^\times$ the Teichmüller character (a homomorphism such that $\zeta_1^g=\zeta_1^{\chi(g)}$ for all $g\in\Delta$). We define

$$e_i=\frac{1}{\delta}\sum_{g\in\Delta}\chi(g)^i g^{-1}\in\boldsymbol{Z}_p[\Delta]$$

for each integer $i$. The main purpose of this paper is to prove the following:

**Theorem.** *Let $F$ be an imaginary quadratic field, $p$ an odd prime number, $F_\infty$, $\zeta_n$, $k$, $\Delta$ and $e_i$ as above. Let $k^+$ be the maximal real subfield of $k$, $A^+$ the $p$-primary part of the ideal class group of $k^+$ and $S_0$ the set of all prime ideals of $F$ each of which has only one prime factor in $k(\zeta_2)$. We*

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