# 57. *A Reduction of Hamiltonian Systems with Multi-time Variables Along a Regular Singularity*

By Hironobu KIMURA,*) Atusi MATUMIYA,**) and Kyoichi TAKANO**)

(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 12, 1991)

**1. Introduction.** Let $(t, x)=(t_1, \cdots, t_N, x_1, \cdots, x_{2n})$ be the coordinates of $\mathbf{C}^{2n+N}$ and let $D(r, \rho)$ be an unbounded domian in $\mathbf{C}^{2n+N}$ defined by

$$D(r, \rho):=\{(t, x)\in \mathbf{C}^{2n+N};\, |t|<r, |x_1x_{n+1}|, |t_1x_{n+1}|, |x_i|<\rho, (i\neq n+1)\}$$

where $|a|:=\max\{|a_1|, \cdots, |a_m|\}$ for $a=(a_1, \cdots, a_m)\in \mathbf{C}^m$. The projection image to $D(r, \rho)$ to the $t$-space is a polydisk with center 0, which we denote by $\Delta(r):=\{t\in \mathbf{C}^N;\, |t|<r\}$. The domain $D(r, \rho)$ is a neighbourhood of (0, 0).

Consider a completely integrable Hamiltonian system of the form:

$$(1)\qquad t_i\partial_i x=JH^i_x,\qquad J=\begin{pmatrix} 0 & I_n \\ -I_n & 0\end{pmatrix},\qquad 1\leq i\leq N$$

with Hamiltonians $H^1, \cdots, H^N$ holomorphic in $D(r, \rho)$, where $\partial_i=\partial/\partial t_i$ and $H^i_x:={}^t(H^i_{x_1}, \cdots, H^i_{x_{2n}})$ is the gradient vector of $H^i$ in $x$. The system (1) is said to have *a singularity of regular type along a hyperplane* $S:=\{t\in\Delta(r);\, t_1=0\}$, if $H^i/t_i$ $(2\leq i\leq N)$ are holomorphic in $D(r, \rho)$ and if $H^1$ does not have $t_1$ as a factor.

The purpose of this note is to obtain a reduction theorem for the system (1) with a singularity of regular type along $S$ (Theorem 1). This result will be applied to the Hamiltonian system $\mathcal{H}_n$ (see § 2) which is a generalization of the sixth Painlevé system [7] to a system of partial differential equations obtained by a monodromy preserving deformation.

We say that a symplectic transformation $\phi: (t, x)\to(t, X)$ is *♯-symplectic* if $\phi$ is holomorphic on $D(r, \rho)$ and if $D(r', \rho')\subset\phi(D(r, \rho))$ for some positive $r'$ and $\rho'$.

We define a class of Hamiltonians studied in this note. Consider a Hamiltonian system (1) with a Hamiltonian $\mathbf{H}=(H^1, \cdots, H^N)$. We expand $H^i$ in $x$ as

$$H^i(t, x)={}^tH^i_x(t, 0)x+\frac{1}{2}{}^txH^i_{xx}(t, 0)x+\sum_{\substack{\alpha+e_1+e_{n+1}\geq 0\\ |\alpha+e_1+e_{n+1}|\geq 3}} h^i_\alpha(t)x^{\alpha+e_1+e_{n+1}}$$

for $1\leq i\leq N$, where $H^i_{xx}$ denotes the Hessian of $H^i$ with respect to $x$ and $x^{\alpha+e_1+e_{n+1}}=x_1^{\alpha_1+1}\cdots x_n^{\alpha_n}x_{n+1}^{\alpha_{n+1}+1}\cdots x_{2n}^{\alpha_{2n}}$.

We assume the following four conditions:

(A-1) $H^1, H^2/t_2, \cdots, H^N/t_N$ are bounded holomorphic functions in $D(r, \rho)$.

(A-2) $H^1$ satisfies

*) Department of Mathematics, College of Arts and Sciences, University of Tokyo.
**) Department of Mathematics, Faculty of Sciences, Kobe University.