54. A Note on Multivalent Functions

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1. Introduction. Let $A_p(n)$ be the class of functions of the form

$$f(z)=z^p+\sum_{k=p+n}^{\infty} a_k z^k \quad (p\in N=\{1,2,3,\cdots\};\ n\in N) \tag{1.1}$$

which are analytic in the open unit disk $U=\{z:|z|<1\}$. A function $f(z)\in A_p(n)$ is said to be in the class $A_p(n,\alpha)$ if it satisfies

$$\left|\frac{f(z)}{z^p}-1\right|<1-\alpha \tag{1.2}$$

for some $\alpha(0\leq\alpha<1)$ and for all $z\in U$.

Recently, Saitoh [3] has studied the class $A_p(n,\alpha)$ and proved some properties for functions belonging to $A_p(n,\alpha)$. Our main result in this paper contains a result due to Saitoh [3, Theorem 1].

2. Main result. We derive the main result by using the following lemma due to Miller and Mocanu [2] (also, due to Jack [1]).

Lemma. *Let $w(z)=w_n z^n+w_{n+1}z^{n+1}+\cdots$ be regular in U with $w(z)\not\equiv 0$ and $n\geq 1$. If $z_0=r_0e^{i\theta_0}$ $(r_0<1)$ and*

$$|w(z_0)|=\sum_{|z|\leq r_0}|w(z)| \tag{2.1}$$

then

$$z_0w'(z_0)=mw(z_0) \tag{2.2}$$

and

$$\mathrm{Re}\left(1+\frac{z_0w''(z_0)}{w'(z_0)}\right)\geq m, \tag{2.3}$$

where $m\geq n\geq 1$.

Theorem. *If $f(z)\in A_p(n)$ with $f(z)\not\equiv z^p$ satisfies*

$$\left|\beta\frac{f(z)}{z^p}+\gamma\frac{f'(z)}{z^{p-1}}-(\beta+p\gamma)\right|<(1-\alpha)\{\beta+(p+n)\gamma\} \tag{2.4}$$

for some $\alpha(0\leq\alpha<1)$, $\beta(\beta\geq 0)$, $\gamma(\gamma\geq 0)$, $\beta+\gamma>0$, and for all $z\in U$, then $f(z)\in A_p(n,\alpha)$.

Proof. Defining the function $w(z)$ by

$$\frac{f(z)}{z^p}-1=(1-\alpha)\omega(z) \tag{2.5}$$

for $f(z)\in A_p(n)$, we see that $w(z)=w_nz^n+w_{n+1}z^{n+1}+\cdots$ is regular in U and $w(z)\not\equiv 0$. It follows from (2.5) that

$$\frac{f'(z)}{z^{p-1}}=p+(1-\alpha)\{pw(z)+zw'(z)\}. \tag{2.6}$$

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