52. *Centralizers of Galois Representations in Pro-l Pure Sphere Braid Groups*

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(Communicated by Shokichi IYANAGA, M. J. A., June 11, 1991)

The purpose of this note is to announce a result about exterior Galois representations in pro-l pure sphere braid groups by summarizing [9, 10]. Let $M_{0,n}$ be the moduli variety of the isomorphism classes of ordered n-pointed projective lines considered to be defined over a number field k, and let l be a rational odd prime. Then we have an exact sequence of profinite groups

$$(*) \qquad 1 \longrightarrow \Gamma_0^{n,\text{pro-}l} \longrightarrow \pi_1^{(l)}(M_{0,n}) \longrightarrow G_k \longrightarrow 1,$$

where

$$\begin{cases} G_k = \text{the absolute Galois group of } k, \\ \Gamma_0^{n,\text{pro-}l} = \text{the pro-}l \text{ completion of } \hat{\Gamma}_0^n := \pi_1(M_{0,n} \otimes \bar{k}), \\ \pi_1^{(l)}(M_{0,n}) = \pi_1(M_{0,n}) / \ker(\hat{\Gamma}_0^n \longrightarrow \Gamma_0^{n,\text{pro-}l}). \end{cases}$$

Our main result can be stated as follows.

Theorem 1. *Let $\varphi_n : G_k \to \operatorname{Out} \Gamma_0^{n,\text{pro-}l}$ be the exterior Galois representation induced from the exact sequence (*). Then the centralizer of the Galois image $\varphi_n(G_k)$ in $\operatorname{Out} \Gamma_0^{n,\text{pro-}l}$ is isomorphic to S_3 when $n=4$, and to S_n when $n \geq 5$. Here S_n denotes the symmetric group of degree n.*

We say that an automorphism of $\pi_1^{(l)}(M_{0,n})$ is *Galois equivariant* if it induces identity on the quotient $G_k = \pi_1^{(l)}(M_{0,n}) / \Gamma_0^{n,\text{pro-}l}$.

Theorem 2. *Let $E_k(M_{0,n})$ denote the quotient of the group of Galois equivariant automorphisms of $\pi_1^{(l)}(M_{0,n})$ modulo the inner automorphisms by $\Gamma_0^{n,\text{pro-}l}$, and let $\operatorname{Aut}_k M_{0,n}$ be the k-automorphism group of the variety $M_{0,n}$. Then the canonical homomorphism*

$$\Phi_n : \operatorname{Aut}_k M_{0,n} \longrightarrow E_k(M_{0,n})$$

gives a bijection.

As $\operatorname{Aut}_k M_{0,n}$ for $n \geq 5$ is known to be isomorphic to S_n [13], we see that Theorem 2 is a restatement of Theorem 1 by an argument of (profinite) group theory. Theorem 2 for $M_{0,4} = \boldsymbol{P}^1 - \{0, 1, \infty\}$ is proved in [8] as an application of Ihara's theory [4], Belyi's lifting [1], and a characterization of inertia groups in terms of nonabelian weight filtration [7]. For $n \geq 5$, it is shown by N. V. Ivanov [6] that the exterior automorphism group of the discrete group $\Gamma_0^n := \pi_1(M_{0,n}(\boldsymbol{C}))$ is finite. Our theorem asserts that, while the pro-l completion $\Gamma_0^{n,\text{pro-}l}$ has infinite (nonabelian) exterior Galois symmetries by Ihara's result [5], the exterior *Galois equivariant* symmetries of it are again limited to being finite. This will support a conjecture about the arithmetic analogue of the Ivanov-McCarthy rigidity of