6. On a Determination of Real Quadratic Fields of Class Number One and Related Continued Fraction Period Length Less than 25

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§1. **Introduction.** The primary thrust of this paper is to investigate real quadratic fields $Q(\sqrt{d})$ of class number $h(d)$ equal to 1 when related to the period length, k of the continued fraction expansion of ω where $\omega=(\sigma-1+\sqrt{d})/\sigma$ with $\sigma=\begin{Bmatrix}1 \text{ if } d\equiv 2,3 \pmod 4\\ 2 \text{ if } d\equiv 1 \pmod 4\end{Bmatrix}$. We actually determine, (with only one possible value remaining, whose very existence would be a counterexample to the Riemann hypothesis), all those positive square-free integers d with $h(d)=1$ and $k\leq 24$. Moreover our new approach allows us to reformulate the Gauss conjecture as to the infinitude of real quadratic fields $K=Q(\sqrt{d})$ with d positive square-free and $h(d)=1$, in terms of the theory of continued fractions.

§2. **Notations and preliminaries.** We let $\mathcal{O}_K$ denote the maximal order in $K=Q(\sqrt{d})$.

Throughout d will be assumed to be a positive square-free integer. For convenience sake we collect together basic facts involving continued fractions which we will be using throughout the paper.

For ω as above let the continued fraction expansion of ω be denoted by $\omega=\langle a,\overline{a_1,\cdots,a_k}\rangle$. Then $a_0=a=\lfloor\omega\rfloor$, $a_i=\lfloor(P_i+\sqrt{d})/Q_i\rfloor$ for $i\geq 1$ (here $\lfloor\ \rfloor$ denotes the greatest integer function), where $(P_0, Q_0)=(1,2)$ if $d\equiv 1 \pmod 4$ and $(P_0,Q_0)=(0,1)$ otherwise. Also,

(2.1) $P_{i+1}=a_iQ_i-P_i$ for $i\geq 0$,

(2.2) $Q_{i+1}Q_i=d-P_{i+1}^2$ for $i\geq 0$, and

(2.3) $a_i=a_{k-i}$ for $1\leq i\leq k-1$.

Moreover either,

(2.4) $P_i=P_{i+1}$ in which case $k=2i$ or

(2.5) $Q_i=Q_{i+1}$ in which case $k=2i+1$.

Now we give some background to the research involved herein. In [2] Kim and Leu showed that 2 conjectures (one of Chowla [1], and one of Yokoi [15]) are valid with one possible exceptional value remaining, and therefore that one of the 2 conjectures is valid with the remaining one failing for at most one value. In [7] we proved Chowla's conjecture under the assumption of the generalized Riemann hypothesis (GRH). Subsequently we extended

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