5. A Necessary Condition for Monotone (P, μ)-u.d. mod 1 Sequences

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Abstract: Schatte [2: assertion (15)] remarked that

$$\lim_{n\to\infty} g(n)/\log n=\infty,$$

if the sequence $(g(n))$ is non-decreasing and uniformly distributed in the ordinary sense. Niederreiter proved ([1] Theorem 2) that:

Let μ be a Borel probability measure on R/Z that is not a point measure and let p be a weighted means. If $(g(n))$ is a non-decreasing (P, μ)-u.d. mod 1 sequence, then necessarily

$$(*) \qquad \lim_{n\to\infty} g(n)/\log s(n)=\infty,$$

where $s(n)=p(1)+p(2)+\cdots+p(n)$ is such that $s(n)\uparrow\infty$.

In this paper we shall prove $(*)$ along the same lines as Schatte.

§1. **Definitions.** Let $P=(p(n))$, $n=1, 2, \cdots$, be a sequence of non-negative real numbers with $p(1)>0$. For $N\geqq 1$, we put $s(N)=p(1)+p(2)+\cdots+p(N)$ and assume throughout that $s(N)\to\infty$ as $N\to\infty$.

We define after Tsuji [3] the $(M, p(n))$-u.d. mod 1.

Definition 1. A sequence $(g(n))$ is said to be $(M, p(n))$-uniformly distributed mod 1 (or shortly $(M, p(n))$-u.d. mod 1), if

$$(1) \qquad \lim_{N\to\infty}\frac{1}{s(N)}\sum_{n=1}^{N} p(n)C_J(\{g(n)\})=|J|,$$

holds for all intervals J in $\boldsymbol{R}/\boldsymbol{Z}$. Here C_J denotes the characteristic function of J.

It is known that an alternative definition is as follows:

A sequence $(g(n))$ is said to be $(M, p(n))$-u.d. mod 1 if for all positive integers h,

$$\lim_{N\to\infty}\frac{1}{s(N)}\sum_{n=1}^{N} p(n)e^{2\pi i h g(n)}=0.$$

We define after Niederreiter [1] the (P, μ)-u.d. mod 1 as follows:

Definition 2. Let $(p(n))$ and $(s(n))$ be sequences of Definition 1 and μ be a Borel probability measure on $\boldsymbol{R}/\boldsymbol{Z}$. Then a sequence $(g(n))$ is said to be (P, μ)-uniformly distributed mod 1 (or shortly (P, μ)-u.d. mod 1), if

$$(2) \qquad \lim_{N\to\infty}\frac{1}{s(N)}\sum_{n=1}^{N} p(n)C_J(\{g(n)\})=\mu(J),$$

holds for all J in $\boldsymbol{R}/\boldsymbol{Z}$. Or equivalently, a sequence $(g(n))$ is said to be (P, μ)-u.d. mod 1 if

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