41. On Non-stationary Boussinesq Equations

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Let Ω be a bounded domain in $R^2(2\leq n\leq 4)$, the boundary of which satisfies the next condition.

Condition (H). *$\partial\Omega$ is of class C^1 and divided as follows: $\partial\Omega=\Gamma_1\cup\Gamma_2$, $\Gamma_1\cap\Gamma_2=\phi$, measure of $\Gamma_1\neq 0$, and the intersection $\bar{\Gamma}_1\cap\bar{\Gamma}_2$ is an $n-2$ dimensional C^1 manifold.*

We consider the following initial boundary value problem:

$$(1)\qquad \begin{cases} \dfrac{\partial u}{\partial t}+(u\cdot\nabla)u=-\dfrac{1}{\rho}\nabla p+\nu\Delta u+\beta g\theta, & \\ \operatorname{div} u=0, & x\in\Omega,\ t>0, \\ \dfrac{\partial\theta}{\partial t}+(u\cdot\nabla)\theta=\chi\Delta\theta, & \end{cases}$$

$$(2)\qquad \begin{cases} u(x,t)=0,\ \theta(x,t)=\xi(x,t), & x\in\Gamma_1,\ t>0, \\ u(x,t)=0,\ \dfrac{\partial}{\partial n}\theta(x,t)=\eta(x,t), & x\in\Gamma_2,\ t>0, \end{cases}$$

$$(3)\qquad \begin{cases} u(x,0)=a_0(x), & x\in\Omega, \\ \theta(x,0)=\tau_0(x), & \end{cases}$$

where $u=(u_1, u_2, \cdots, u_n)$ is the fluid velocity, p is the pressure, θ is the temperature, $u\cdot\nabla=\sum_{j=1}^n u_j(\partial/\partial x_j)$, $(\partial\theta/\partial n)$ denotes the outer normal derivative of θ at x to $\partial\Omega$, $g(x,t)$ is the gravitational vector function, and ρ(density), ν(kinematic viscosity), β(coefficient of volume expansion), χ(thermal diffusivity) are positive constants. $\xi(x,t)$ (resp. $\eta(x,t)$) is a function defined on $\Gamma_1\times(0,T)$ (resp. $\Gamma_2\times(0,T)$) and $a_0(x)$ (resp. $\tau_0(x)$) is a vector (resp. scalar) function defined on Ω. This system of equations (1) describes the motion of fluid of heat convection (Boussinesq approximation).

In our previous papers [7, 8], we showed the existence of weak solution of the stationary problem. In this paper, we report the existence of a weak solution of evolutional problem (1), (2), (3) (Theorem 1), its uniqueness and some regularity property (Theorems 2, 3), and the existence of solutions with reproductive property (Theorem 4).

Firstly we define some function spaces. The functions considered in this paper are all real valued. $L^p(\Omega)$ and the Sobolev space $W_p^l(\Omega)$ are defined as usual. We also denote $H^l(\Omega)=W_2^l(\Omega)$. Whether the elements of space are scalar or vector functions is understood from the contexts unless stated explicitly.

The solenoidal function spaces are as follows:

$D_\sigma=\{$vector function $\varphi\in C^\infty(\Omega)\,|\,\operatorname{supp}\varphi\subset\Omega,\ \operatorname{div}\varphi=0$ in $\Omega\}$,