40. *Merger of Chaotic Bands in Period-doubling Cascades*

By Minako YOSHIMOTO and Kiyoko NISHIZAWA
Department of Mathematics, Faculty of Science and Technology,
Sophia University

(Communicated by Shokichi IYANAGA, M. J. A., May 13, 1991)

1. Introduction. One-dimentional recursion relation $x_{n+1}=F(x_n, \lambda)$, where $F(x, \lambda)$ is a real-valued function of real variables x and λ, which is smooth in λ, provided an important example as to the nature of the onset of chaos in dissipative dynamical systems. There has been much effort devoted to describing the properties of these maps. The sequence of period doubling bifurcations and its universal scaling behaviour have received particularly intense investigation.

Feigenbaum studied the quadratic one-dimentional maps (cf. [2])

$$f_\lambda(x)=\lambda x(1-x), \qquad 1\leq\lambda\leq 4.$$

When $\lambda<3$, successive iterations $x_{n+1}=\lambda x_n(1-x_n)$ converge to a point attractor of period one. As λ is increased up to the critical parameter value λ_∞ ($\sim$3.5700), known as Feigenbaum point, one obtains period-doubling bifurcations leading to period-2^n orbits. It is not known whether Feigenbaum point λ_∞ is rational or irrational number. See Fig. 1. When one

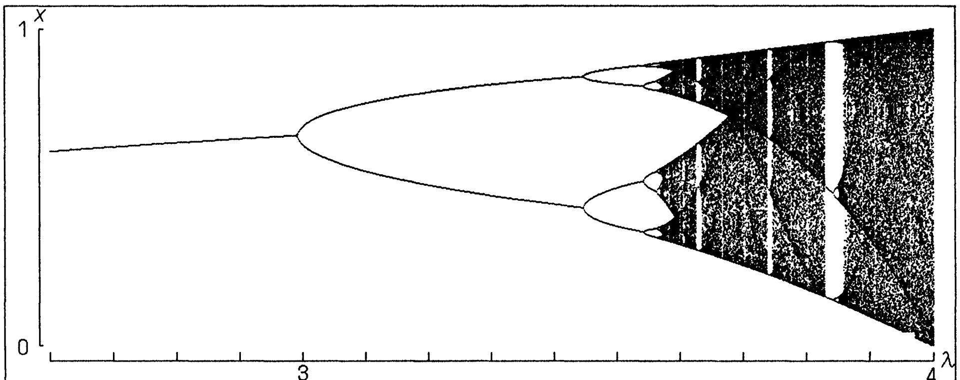

Fig. 1. Bifurcation diagram for the system of equation $f_\lambda(x)=\lambda x(1-x)$.

Fig. 1. Merger of chaotic bands.

periodic orbit turns to be unstable, as we increase λ, another stable orbit is created, with twice the period of the first one. The new orbit itself turns to be unstable at a larger value of λ and another stable orbit is created, with four times the period of the first one, etc. Thus a period-2^k orbit bifurcates, at λ_k, from a period-2^{k-1} orbit. The resulting 'Feigenbaum Sequence' $\{\lambda_k\}$, converges increasingly to the critical value λ_∞, with

$$\lim_{k\to\infty}\frac{\lambda_k-\lambda_{k-1}}{\lambda_{k+1}-\lambda_k}=\delta_F.$$