4. *A Remark on Free Boundary Plateau Problem for Surfaces of General Topological Type**)

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The classical Plateau problem asks for a disk type surface of least area spanning a prescribed Jordan curve in R^3. It was independently solved by Douglas and Radó in 1931. Douglas [3] considered also the case for surfaces of general topological type or surfaces of any genus with any number of boundary components, and solved it under a so called "Douglas' condition". Courant derived a different proof using not Douglas functional, but Dirichlet integral. (See Courant [2].) He also gave another notion or "the condition of cohesion", which guarantees the existence of solutions. Courant's proof was restricted to the case of surfaces of genus 0, but Shiffman [8] proved it for general case. Recently Tomi-Tromba [9] presented a modern approach to Shiffman's result, using Teichmüller space.

In this short note, we remark that a free boundary version can be described along this line, under what we call "the condition of inter-border cohesion", in addition to the condition of cohesion. In the author's paper [6], we utilized "boundary incompressibility". This qualitative notion corresponds to the quantitative one or the condition of inter-border cohesion, while the incompressibility corresponds to the condition of cohesion.

Free boundary Plateau problem. Let M be a compact smooth surface with boundary components $C_1, \cdots, C_k$. Let S be a compact smooth submanifold of R^n, on which all our free boundaries should lie. We give here a class of "surfaces (or mappings) with free boundary on S" as follows:

$$\mathcal{F} := C^0(M, R^n) \cap W^{1,2}(M, R^n) \cap \{f ; f(C_j) \subset S\},$$

where $W^{1,2}(M, R^n)$ denotes the ordinary Sobolev space of L^2-elements whose first derivatives (in the sense of distributions) belong also to L^2. We want to minimize the area

$$A(f) := \int_M \|df \wedge df\|,$$

in this class $\mathcal{F}$. However the infimum may be attained in general only by maps from the surfaces of lower topological types. So we recall the following condition for non-degeneracy (see Courant [2], p. 145, Tomi-Tromba [9], p. 51). This condition can be expressed, at least formally, in terms concerning only the present class, while Douglas' condition needs an information of the infimum of area in the class of all degenerate maps.

*) Dedicated to Professor Tatsuo Fuji'i'e on his 60th Birthday.