34. Weinstein Conjecture and a Theory of Infinite Dimensional Cycles

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Introduction. Let (M, ω) be a contact manifold of dimension $2n+1$. Then there exists on M a vector field ξ, called a characteristic field (or Reeb field) such that

$$d\omega(\cdot, \xi) \equiv 0,$$
$$\omega(\xi) \equiv 1.$$

If M is an imbedded star-shaped sphere in $\boldsymbol{R}^{2n+2}$, and if f is a smooth function on $\boldsymbol{R}^{2n+2}$ such that $M = f^{-1}(k)$ for some $k \in \boldsymbol{R}$ and df is nowhere zero on M, then ξ is a Hamiltonian vector field of f with respect to the canonical symplectic structure Ω on $\boldsymbol{R}^{2n+2}$ (after a normalization). A. Weinstein [5] and P. Rabinowitz [4] showed there exists at least one closed orbit of ξ for any star-shaped sphere. In view of this result, the existence of closed orbits of ξ for any compact contact manifolds was conjectured by A. Weinstein.

For compact hypersurfaces of contact type in $\boldsymbol{R}^{2n+2}$, the conjecture was solved affirmatively by Viterbo [6]. His result was extended by Floer, Hoffer and Viterbo [2] for compact hypersurfaces of contact type in $\boldsymbol{C}^l \times P$, here (P, Ω) is a compact symplectic manifold, $l>0$ and Ω is supposed to vanish on $\pi_2(P)$.

This problem has the following variational aspect. Closed orbits of ξ coincide with the critical points of the following variational problem:

$$L(c) = \int \omega(\dot{c})ds$$
$$c \in C^1(S^1, M)$$

A neck of solving the conjecture for a general case lies in a break-down of the so calld Palais-Smale condition. This leads us to the notion of *critical points at infinity*, which are defined to be the set of *limit points* of sequences c_i such that the action of c_i tends to zero. In this paper we discuss this failure of the Palais-Smale condition and identify these critical points at infinity, using a theory of infinite dimensional cycles.

We define in the next section a family of operators $P = \{P_c\}$ parametrized by a free loop space $C^1(S^1, M)$. We derive from this family of operators a number of infinite dimensional cycles in the space $C^1(S^1, M)$. A general theory of infinite dimensional cycles associated to operators was studied in [3], to which we refer for notations of cycles. Among these cycles, our interest lies in a solution cycle $\kappa^{1,1}(P)$.