32. *On Some Discrete Reflection Groups and Congruence Subgroups*[*)]

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In this paper we present some congruence subgroups of the groups of automorphisms of the balls and the classical bounded domains of type IV and announce that they are generated by finitely many reflections. The details and the relation between these groups and the differential equations will be given in the forthcoming paper [3, 4].

1. Let A be a Hermitian form with signature $(n-,1+)$ on an $(n+1)$-dimensional vector space V; the form $(u,v):=A(u,v)$ is supposed to be $\boldsymbol{C}$-linear in v and anti-$\boldsymbol{C}$-linear in u. Let

$$V^+=\{v\in V\,|\,(v,v)>0\},\quad V^0=\{v\in V\,|\,(v,v)=0\},\quad V^-=\{v\in V\,|\,(v,v)<0\}.$$

Notice that $\boldsymbol{D}:=V^+/\boldsymbol{C}^\times$ is isomorphic to the unit ball $\{(z_1,\cdots,z_n)\in\boldsymbol{C}^n\,|\,|z_1|^2+\cdots+|z_n|^2<1\}$ and that the group $Aut(\boldsymbol{D})$ of automorphisms of $\boldsymbol{D}$ is given by the projectivization of the group $\{g\in GL(V)\,|\,(gu,gv)=(u,v)\}$. Notice also that $\partial\boldsymbol{D}=V^0/\boldsymbol{C}^\times$. For $\alpha\in V^-$, we define the following transformation R_α called the reflection with respect to a root α by

$$v\mapsto v-2\frac{(\alpha,v)}{(\alpha,\alpha)}\alpha,$$

which keeps the Hermitian form A invariant; in particular, it defines an automorphism of $\boldsymbol{D}$. Notice that R_α pointwisely keeps the subspace $\alpha^\perp=\{v\in V\,|\,(\alpha,v)=0\}$, which is called the mirror of the reflection. A group generated by reflections is called a reflection group.

Let $n=5$ and fix a basis of the linear space V and the Hermitian form $A=(a_{jk})$ $(1\le j,k\le 6)$ as follows:

$$a_{jj}=-2,\quad a_{jk}=\bar{a}_{kj}=-1+i\ (j<k),\quad \text{where } i=\sqrt{-1}.$$

Let $\boldsymbol{Z}[i]$ be the ring of Gauss integers; the full modular group Γ is defined by

$$\Gamma=\{X\in GL(6,\boldsymbol{Z}[i])\,|\,X^*AX=A\}.$$

The principal congruence subgroup $\Gamma(1-i)$ with respect to the ideal $(1-i)\subset\boldsymbol{Z}[i]$ is defined by

$$\Gamma(1-i)=\{X\in\Gamma\,|\,X\equiv I_6 \bmod (1-i)\}.$$

An integral root of norm -2 is a vector $\alpha\in V^-$ whose entries are in $\boldsymbol{Z}[i]$ such that $(\alpha,\alpha)=-2$. By definition, for every integral root α of norm -2, the reflection R_α belongs to $\Gamma(1-i)$.

Theorem 1. (1) *The group $\Gamma(1-i)$ is generated by finitely many reflections with respect to integral roots of norm -2, e.g.*

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*) Dedicated to Professor Bernard Morin on his 60th birthday.