31. Minimal Quasi-ideals in Abstract Affine Near-rings. II

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1. Introduction. In ring theory, it is well known that each one of the intersection and the product of a minimal right ideal and a minimal left ideal of a ring is either $\{0\}$ or a minimal quasi-ideal of the ring (see [2]). In [5], this result has been generalized for zero-symmetric near-rings.

The purpose of this note is to extend the above result to a class of abstract affine near-rings. For the basic terminology and notation we refer to [1].

2. Preliminaries. Let N be a near-ring, which always means right one throughout this note.

If A and B are two non-empty subsets of N, then AB denotes the set of all finite sums of the form $\sum a_k b_k$ with $a_k \in A$, $b_k \in B$, and $A*B$ denotes the set of all finite sums of the form $\sum (a_k(a'_k+b_k)-a_k a'_k)$ with $a_k, a'_k \in A$, $b_k \in B$.

A right ideal of N is a normal subgroup R of $(N, +)$ such that $RN \subseteq R$, and a left ideal of N is a normal subgroup L of $(N, +)$ such that $N*L \subseteq L$. A quasi-ideal of N is a subgroup Q of $(N, +)$ such that $N*Q \cap NQ \cap QN \subseteq Q$. Right ideals and left ideals are quasi-ideals. The intersection of a family of quasi-ideals is again a quasi-ideal.

A non-zero quasi-ideal Q of N is minimal if the only quasi-ideal of N contained in Q are $\{0\}$ and Q. Similarly, one defines minimal right ideals and minimal left ideals.

A near-ring N is called an abstract affine near-ring if N is abelian and $N_0 = N_d$, where N_0 and N_d are the zero-symmetric part and the set of all distributive elements of N, respectively.

Let N be an abstract affine near-ring. Then the following hold (see [3] and [4]):

(a) A subgroup L of $(N, +)$ is a left ideal of N if and only if $N_0 L \subseteq L$.

(b) If S is a subgroup of $(N, +)$, then $N_0 S$ is a left ideal of N and SN is a right ideal of N.

(c) A subgroup Q of $(N, +)$ is a quasi-ideal of N if and only if $N_0 Q \cap QN \subseteq Q$.

3. Main results. We start with

Lemma 1. *Let N be an abstract affine near-ring. Then a minimal right (left) ideal of N contained in N_0 is a minimal right (left) ideal of N_0.*

Proof. Let R be a minimal right ideal of N contained in N_0. By [1, Proposition 9.73] we have $R = R_0 + R_c$, where $R_0 = R \cap N_0$ is a right ideal of