28. Universal R-matrices for Quantum Groups Associated to Simple Lie Superalgebras

By Hiroyuki YAMANE
Department of Mathematics, Osaka University

(Communicated by Shokichi IYANAGA, M. J. A., April 12, 1991)

Introduction. Let H be a Hopf algebra with coproduct $\Delta: H \to H \otimes H$. Let $\mathcal{R} = \sum_i a_i \otimes b_i \in H \otimes H$ be an invertible element. The triple $(H, \Delta, \mathcal{R})$ is called a *quasi-triangular Hopf algebra* if $\mathcal{R}$ satisfies the following properties (see [1]):

(0.1) $$\bar{\Delta}(x) = \mathcal{R}\Delta(x)\mathcal{R}^{-1} \qquad (x \in H),$$
$$(\Delta \otimes id)(\mathcal{R}) = \mathcal{R}_{13}\mathcal{R}_{23}, \qquad (id \otimes \Delta)(\mathcal{R}) = \mathcal{R}_{13}\mathcal{R}_{12}$$

where $\bar{\Delta} = \tau \circ \Delta$, $\tau(x \otimes y) = y \otimes x$ and $\mathcal{R}_{12} = \sum_i a_i \otimes b_i \otimes 1$, $\mathcal{R}_{13} = \sum_i a_i \otimes 1 \otimes b_i$, $\mathcal{R}_{23} = \sum_i 1 \otimes a_i \otimes b_i$. The $\mathcal{R}$ is called the *universal R-matrix*. From this definition, it follows that $\mathcal{R}$ satisfies the Yang-Baxter equation:

(0.2) $$\mathcal{R}_{12}\mathcal{R}_{13}\mathcal{R}_{23} = \mathcal{R}_{23}\mathcal{R}_{13}\mathcal{R}_{12}.$$

Let $\mathcal{G}$ be a complex simple Lie algebra and $U(\mathcal{G})$ the universal enveloping algebra of $\mathcal{G}$. In 1985, Drinfeld [1] and Jimbo [2] associated to each $\mathcal{G}$, the h-adic topologically free $C[[h]]$-Hopf algebra $(U_h(\mathcal{G}), \Delta)$ such that $U_h(\mathcal{G})/hU_h(\mathcal{G}) = U(\mathcal{G})$, which is now called the *quantum group* or the *quantized enveloping algebra*. Moreover Drinfeld [1] gave a method of constructing an element $\mathcal{R} = U_h(\mathcal{G}) \hat{\otimes} U_h(\mathcal{G})$ such that $(U_h(\mathcal{G}), \Delta, \mathcal{R})$ is a quasi-triangular Hopf algebra. His method is called the *quantum double construction*. By using this method, Rosso [9] gave an explicit formula of $\mathcal{R}$ for $\mathcal{G} = sl_n(C)$, and Kirillov-Reshetikhin [6], Levendorskii-Soibelman [8] gave such a formula for any $\mathcal{G}$.

Let $\tilde{\mathcal{G}} = \tilde{\mathcal{G}}_0 \oplus \tilde{\mathcal{G}}_1$ be a complex simple Lie superalgebra of types $A-G$ and $U(\tilde{\mathcal{G}})$ the universal enveloping superalgebra of $\tilde{\mathcal{G}}$. In this note, we associate to each $\tilde{\mathcal{G}}$, an h-adic topological $C[[h]]$-Hopf superalgebra $(U_h(\tilde{\mathcal{G}}), \Delta^s)$ such that $U_h(\tilde{\mathcal{G}})/hU_h(\tilde{\mathcal{G}}) = U(\tilde{\mathcal{G}})$. In fact, the definition of $U_h(\tilde{\mathcal{G}})$ depends on a choice of the Cartan matrix and the parities of the simple roots of $\tilde{\mathcal{G}}$. (For the terminologies *Lie superalgebra* and *Hopf superalgebra*, see [4, 6].) We also introduce an h-adic topological Hopf algebra $(U_h^\sigma(\tilde{\mathcal{G}}), \Delta^\sigma)$. The $U_h^\sigma(\tilde{\mathcal{G}})$ contains $U_h(\tilde{\mathcal{G}})$ as a subalgebra and the Hopf algebra structure of $(U_h^\sigma(\tilde{\mathcal{G}}), \Delta^\sigma)$ arises naturally from the Hopf superalgebra structure of $(U_h(\tilde{\mathcal{G}}), \Delta^s)$. In this note, by using the quantum double construction, we construct an element $\mathcal{R} \in U_h^\sigma(\tilde{\mathcal{G}}) \otimes U_h^\sigma(\tilde{\mathcal{G}})$ explicitly so that $(U_h^\sigma(\tilde{\mathcal{G}}), \Delta^\sigma, \mathcal{R})$ is a quasi-triangular Hopf algebra. In the process of constructing $\mathcal{R}$, we can also show that $U_h^\sigma(\tilde{\mathcal{G}})$ and $U_h(\tilde{\mathcal{G}})$ are topologically free.

Details omitted here will be published elsewhere.

After I finished this work, Professor E. Date informed me about the