3. *Remarks on the Stability of Certain Periodic Solutions of the Heat Convection Equations*

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§1. **Introduction.** Let $\Omega(t)$ be a time-dependent bounded space domain in R^m ($m=2$ or 3) whose boundary $\partial\Omega(t)$ consists of two components, namely, $\partial\Omega(t)=\Gamma_0\cup\Gamma(t)$. Here Γ_0 is the inner boundary and $\Gamma(t)$ is the outer one. Moreover, these two boundaries do not intersect each other. We denote by K the compact set which is bounded by Γ_0. Let $u=u(x,t)$, $\theta=\theta(x,t)$ and $p=p(x,t)$ be the velocity of the viscous fluid, the temperature and the pressure, respectively. We consider the heat convection equation (HC) of Boussinesq approximation in $\hat{\Omega}=\bigcup_{0<t<T}\Omega(t)\times\{t\}$ with boundary conditions

(1) $\qquad u|_{\partial\Omega(t)}=\beta(x,t),\quad \theta|_{\Gamma_0}=T_0>0,\quad \theta|_{\Gamma(t)}=0$ for any $t\in(0,T)$.

In our previous paper [4], we have proven the unique existence of the time-periodic strong solution of (HC) with (1), provided the domain $\Omega(t)$ and the boundary data $\beta(x,t)$ both vary periodically with period T. The purpose of this paper is to show the asymptotic stability of the periodic solution which is obtained in [4].

§2. **Assumptions and results.** We make some assumptions:

(A1) For any fixed $t>0$, $\Gamma(t)$ and Γ_0 are both simple closed curves (or surfaces) and also they are of class C^3.

(A2) $\Gamma(t)\times\{t\}$ $(0<t<T)$ changes smoothly (say, of class C^4) with respect to t. (See, Assumptions II and III in [4].)

(A3) $g(x)$ is a bounded and continuous vector function in $R^m\setminus\operatorname{int}K$.

(A4) $\beta(x,t)$ is sufficiently smooth in x and t. Moreover, it satisfies the following condition

$$\int_{\partial\Omega(t)}\beta\cdot n\,dS=0,$$

where n is the outer normal vector to $\partial\Omega(t)$.

(A5) The domain $\Omega(t)$ and the function $\beta(x,t)$ vary periodically in t with period $T>0$, i.e., $\Omega(t+T)=\Omega(t)$, $\beta(\cdot,t+T)=\beta(\cdot,t)$ for each $t>0$.

Since $\Omega(t)$ is bounded, there exists an open ball B_1 with radius d such that $\overline{\Omega(t)}\subset B_1$. We put $B=B_1\setminus K$. We introduce a solenoidal periodic function b over B such that $b(x,t)=\beta(x,t)$ on $\partial\Omega(t)$ and an appropriate function $\bar{\theta}$ on $\Omega(t)$ with the same boundary values on $\partial\Omega(t)$ as θ.

We now set the periodicity condition

(2) $\qquad u(\cdot,0)=u(\cdot,T)\qquad$ in $\Omega(0)=\Omega(T)$,

and consider the periodic problem for (HC) with (1) and (2).