14. *Fractal Aspects of Localization of Algebraic Integers and Complex Dynamical Systems*

By Kiyoko NISHIZAWA, Koji SEKIGUCHI, and Kunio YOSHINO
Department of Mathematics, Faculty of Science and Technology,
Sophia University

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1. Introduction. In this paper, we will consider the following problem.

Problem. *Let D be a compact set in $\boldsymbol{C}$. Characterize the properties of algebraic integers contained in D together with all their conjugates.*

Our problem is closely related to the Diophantine moment problem in the ferromagnetic Ising model and the theory of arithmetic holomorphic functions (see [6]).

An answer to our problem has been given by M. Fekete in 1923. He obtained the following theorem.

Theorem 1 (Fekete). *Suppose that D is a compact set in $\boldsymbol{C}$ with transfinite diameter less than* 1. *Then there exist only finitely many algebraic integers which are contained in D together with all their conjugates over the rational number field $\boldsymbol{Q}$.*

If the transfinite diameter is equal to 1, then there exist in general infinitely many such algebraic integers. A typical example is the following celebrated Kronecker's unit theorem (see [3]).

Theorem 2 (Kronecker). *Let D be the unit disc with center* 0 *in $\boldsymbol{C}$. If an algebraic integer is contained in D together with all its conjugates over $\boldsymbol{Q}$, then it is either* 0 *or one of the nth roots of* 1 *for some natural number n.*

For the details of the transfinite diameter, we refer the reader to [2]. For a general compact set D, Problem is still unsolved. But recently from the view point of the theory of complex dynamical systems, it is understood that the localization of algebraic integers is related to the filled-in Julia set of a monic polynomial and to the Mandelbrot set of the monic quadratic polynomials. In 1984, P. Moussa, J.S. Geronimo and D. Bessis obtained the following theorem (see [5]).

Definition 1. *Let T be a polynomial of degree at least* 2. *Then the filled-in Julia set K_T of T is the set of all complex numbers which do not escape to ∞ under the iterations of T.*

Definition 2. *We will denote the nth iterate of T by T^n. A complex number z is said to be a preperiodic point of T, if there exists integers $k>0$ and $l\geq 0$ such that $T^{k+l}(z)=T^l(z)$.*

Theorem 3 (M-G-B). *Let K_T be the filled-in Julia set of a monic polynomial T of degree at least* 2 *with rational integer coefficients. Then the set*