12. Asymptotic Expansion of the Bergman Kernel for Strictly Pseudoconvex Complete Reinhardt Domains in C^2

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(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 13, 1990)

Introduction. The purpose of this note is to present an explicit representation of Fefferman's asymptotic expansion of the Bergman kernel on the diagonal for bounded strictly pseudoconvex complete Reinhardt domains in C^2. As a consequence, we obtain a proof of the so-called Ramadanov conjecture for this class of domains.

Let $K=K(z)$ denote the Bergman kernel restricted to the diagonal of $\Omega\times\Omega$, where Ω is a bounded strictly pseudoconvex domain in C^n with C^∞ boundary. It was shown by C. Fefferman [2] that

$$K=\frac{\varphi}{\lambda^{n+1}}+\psi\log(-\lambda) \qquad \text{with } \varphi,\ \psi\in C^\infty(\bar{\Omega}),$$

where λ is a C^∞ defining function of $\Omega=\{\lambda<0\}$. On the other hand, it is only the ball amongst bounded strictly pseudoconvex domains on which K is explicitly known, and $\psi=0$ in this case. We are thus interested in an explicit example of Ω for which $\psi\neq 0$. The so-called Ramadanov conjecture asks whether Ω is a ball if and only if $\psi=0$ (cf. [4]). A local version of this conjecture was recently solved affirmatively by D. Burns and C. R. Graham in case $n=2$ (cf. [3]).

Assuming in particular that Ω is a complete Reinhardt domain in C^2, we introduce a C^∞ function $p\colon(-\infty, 0]\to R$, which is a hodograph transform describing the boundary of Ω. Then, an asymptotic expansion of K is obtained after a normalization, where the coefficients are expressed in terms of p and its derivatives; furthermore, we can explicitly determine φ mod $O(\lambda^3)$ and $\psi|_{\partial\Omega}$. By using the expression for $\psi|_{\partial\Omega}$, we obtain a proof of the Ramadanov conjecture mentioned above for this class of domains.

The same topic as in the present note was discussed earlier by D. Boichu and G. Coeuré in [1], where they presented an asymptotic expansion of the Bergman kernel in a non-explicit way and attempted to prove the Ramadanov conjecture for the same class of domains as ours. It seems to the present author that their proof of the Ramadanov conjecture is incomplete (Lemme 7 in [1] is incorrect); nevertheless their idea is very useful to us. Namely, we basically follow their method of analysis. The crucial difference is the choice of p, which, together with some other minor modifications, enables us to obtain an explicit expansion.

§1. Statement of the result. Let Ω be a strictly pseudoconvex domain