82. *Some Aspects in the Theory of Representations of Discrete Groups. I**)

By Takeshi HIRAI
Department of Mathematics, Kyoto University

(Communicated by Shokichi IYANAGA, M. J. A., Dec. 12, 1990)

Recently representations of infinite discrete groups are studied by many mathematicians. Here we present some new aspects in this field. First we give general criterions for irreducibility and mutual equivalence for a family of induced representations. Then we apply them to infinite wreath product groups. Further we study in detail induced representations of such groups which are far out of reach of the above criterions. These results will be applied to the infinite symmetric group $\mathfrak{S}_\infty$ to get a big family of completely new irreducible unitary representations (=IURs).

1. Induced vectors. Let G be a discrete group and H a subgroup of G. Take a unitary representation π of H on a space $V(\pi)$ and put $U_\pi=\mathrm{Ind}_H^G\,\pi$. The space $\mathcal{H}(U_\pi)$ for U_π consists of $V(\pi)$-valued functions on G satisfying $f(hg)=\pi(h)f(g)$ $(h\in H,\ g\in G)$, and $\|f\|^2=\sum_{g\in H\backslash G}\|f(g)\|^2<\infty$, where the summation runs over a sections of $H\backslash G$ in G. The representation is given by right translations.

For a vector $v\in V(\pi)$, define an $f\in\mathcal{H}(U_\pi)$ such that $f(e)=v$ and $f(g)=0$ outside of H, where e denotes the unit of G. This f is called the *induced vector* of v and is denoted by $\mathrm{Ind}_H^G\,v$. The set of all induced vectors in $\mathcal{H}(U_\pi)$ is cyclic in the sense that the G-invariant subspace generated by them is everywhere dense. The notion of induced vectors will be necessary to state equivalence relations among IURs constructed here.

2. Boundedness conditions. Let H_1, H_2 be two subgroups of G, and π_i a unitary representation (=UR) of H_i for $i=1,2$. Put $U_{\pi_i}=\mathrm{Ind}_{H_i}^G\,\pi_i$, and let $\mathrm{Hom}_G(U_{\pi_1},U_{\pi_2})=\mathrm{Hom}(U_{\pi_1},U_{\pi_2};G)$ be the space of intertwining operators of U_{π_1} with U_{π_2}. Then, every $T\in\mathrm{Hom}_G(U_{\pi_1},U_{\pi_2})$ is given by a kernel $K(g_2,g_1)$, $g_2,g_1\in G$, with values in $\boldsymbol{B}(V(\pi_1),V(\pi_2))$, the space of bounded linear operators of $V(\pi_1)$ into $V(\pi_2)$, as $(Tf)(g)=\sum_{g'\in H_1\backslash G}K(g,g')f(g')$ $(f\in\mathcal{H}(U_{\pi_1}))$, and the kernel satisfies several conditions (cf. [2]).

For $x\in G$, we put ${}^xg=xgx^{-1}$ $(g\in G)$, $H_2^x=x^{-1}H_2x$ and $(\pi_2^x)(h)=\pi_2({}^xh)$ $(h\in H_2^x)$. Then $L=K(x,e)$ belongs to $\mathrm{Hom}(\pi_1,\pi_2^x;H_1\cap H_2^x)$, and it determines $K(g_2,g_1)$ for $g_2g_1^{-1}\in H_2xH_1$. Furthermore we have for L the following two conditions called the *boundedness conditions*: for a positive constant M,

(B_x) $\sum_{h_1}\|L\pi_1(h_1)v\|^2\leqq M\|v\|^2$ $(h_1\in(H_1\cap x^{-1}H_2x)\backslash H_1)$ for $v\in V(\pi_1)$,

(C_x) $\sum_{h_2}\|L^*\pi_2(h_2)w\|^2\leqq M\|w\|^2$ $(h_2\in(H_2\cap xH_1x^{-1})\backslash H_2)$ for $w\in V(\pi_2)$.

---

*) Dedicated to Professor H. Yoshizawa.