60. *On the Reduction of Binary Cubic Forms with Positive Discriminants. I*

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In a former paper [1], we used the quadruple of integers, named *Voronoi quadruple* (abridged *V-quadruple*), to obtain an integral basis of an order of a cubic field. The same quadruple has been already used by Mathews [2] to develop a theory of reduction of binary cubic forms with negative discriminants. Davenport [3] has given a reduction theory for the case of positive discriminants using another method. In this paper we shall give a reduction theory of binary cubic forms with positive discriminants using the quadruple introduced in [1]. Our main results will be given in § 1. In a subsequent note II, applying this theory and that of Mathews' [2] to the theory of cubic fields, we shall give a method of the construction of a table of non-conjugate cubic fields with discriminants less than a given positive number in absolute value.

§ 1. A binary cubic form

(1) $$f(x, y)=ax^3+bx^2y+cxy^2+dy^3, \quad (a, b, c, d)\in Z^4$$

and another cubic form

(2) $$f'(x, y)=a'x^3+b'x^2y+c'xy^2+d'y^3, \quad (a', b', c', d')\in Z^4$$

are defined to be *equivalent* if there exists a set of integers p, q, r, s which satisfy

(3) $$f'(x, y)=f(px+qy, rx+sy), \quad ps-qr=\pm 1.$$

We express the equivalence as $f\sim f'$ or $(a, b, c, d)\sim(a', b', c', d')$. In such a case, we can write $M=\begin{pmatrix} p & q \\ r & s \end{pmatrix}$, $M\in GL(2, Z)$, and it is easily verified that

$$(a', b', c', d')=(a, b, c, d)\bar{M},$$

where

$$\bar{M}=\begin{bmatrix} p^3 & 3p^2q & 3pq^2 & q^3 \\ p^2r & p(ps+2qr) & q(2ps+qr) & q^2s \\ pr^2 & r(2ps+qr) & s(ps+2qr) & qs^2 \\ r^3 & 3r^2s & 3rs^2 & s^3 \end{bmatrix}\in GL(4, Z).$$

The mapping $\nu: M\to\bar{M}$ gives an injective homomorphism from $GL(2, Z)$ to $GL(4, Z)$ as $\begin{bmatrix} X'^3 \\ X'^2Y' \\ X'Y'^2 \\ Y'^3 \end{bmatrix}=\bar{M}\begin{bmatrix} X^3 \\ X^2Y \\ XY^2 \\ Y^3 \end{bmatrix}$ follows from $\begin{bmatrix} X' \\ Y' \end{bmatrix}=M\begin{bmatrix} X \\ Y \end{bmatrix}$.

The discriminant of the form (1) is the invariant

$$D=b^2c^2-4ac^3-4b^3d+18abcd-27a^2d^2.$$