56. On the Number of Apparent Singularities

By Akihiro SAEKI
Department of Mathematics, Faculty of Science, University of Tokyo

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§0. **Introduction.** Let M be a closed Riemann surface of genus g, S a finite subset of M and ρ a representation of the fundamental group $\pi_1(M-S)$ on C^n. The Riemann-Hilbert problem states as follows:

Find a Fuchsian linear differential equation of order n with singularities on S and having ρ as its monodromy representation.

When S consists of m distinct points, the set of Fuchsian linear differential equations of order n having their singuralities only on S has a structure of a complex manifold of dimension $2^{-1}n(n+1)m+n^2(g-1)$ (Kita [3]), whereas the totality of equivalence classes of complex n-dimensional irreducible representations of the fundamental group $\pi_1(M-S)$ forms a complex manifold of dimension $n^2(m+2g-2)+1$. So, in general, we must introduce apparent singulalities to solve the Riemann-Hilbert problem. On the number of apparent singulalities, the following result is known ([4]):

Theorem 1 (Ohtsuki). *If the representation ρ is irreducible and if the local representation at a point of S induced by ρ is semi-simple, then there exists a Fuchsian linear equation of order n on M which has the given representation ρ as its monodromy representation and has at most*

$$2^{-1}n(n-1)m+n^2(g-1)+1$$

apparent singularities.

In this paper, without assuming that the local representation at a point of S induced by ρ is semi-simple, we show the following theorem:

Main theorem. *Let M, S and ρ be as above Let d be the greatest common divisor of complex dimensions of all invariant subspaces of the local representation induced by ρ at each point of S. Then there exists a Fuchsian linear equation of order n on M which has the given representation ρ as its monodromy representation and has at most*

$$2^{-1}n(n-1)m+n^2(g-1)+d$$

apparent singularities.

Since d is at most n, we have easily the

Corollary. *There exists a Fuchsian linear equation of order n on M which has the given representation ρ as its monodromy representation and has at most*

$$2^{-1}n(n-1)m+n^2(g-1)+n$$

apparent singularities.

§1. **The method of Deligne.** To establish the main theorem, we recall the proof of Theorem 1 (Ohtsuki [4]).