53. *On the Local Zeta Functions of the Hilbert Modular Schemes*

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(Communicated by Shokichi IYANAGA, M. J. A., Sept. 12, 1990)

Abstract: We announce that the local zeta functions of suitable smooth compactifications of Hilbert modular schemes with respect to totally real algebraic number fields K at prime numbers p of good reduction are expressed in terms of the actions of the Hecke rings on their l-adic cohomology groups if p remains prime in K. These actions of the Hecke rings have been discovered in our previous papers: Hatada [10], [11] and [12]. We announce also our estimates for the absolute values of the eigenvalues of those endomorphisms for the Hecke rings of the cohomology groups. Details will appear elsewhere.

§1. **Results.** Let K be a totally real algebraic number field and let $g=[K:\boldsymbol{Q}]$. For simplicity we assume that the strict class number of K is 1, i.e., any non-zero fractional ideal of K is generated by a totally positive element of K. (For example $K=\boldsymbol{Q}(\sqrt{5})$.) We may remove this assumption. Let $\mathcal{R}$ be a commutative ring. Write

$M_{2,2}(\mathcal{R})=$the ring of the 2×2 matrices with coefficients in $\mathcal{R}$;

$\mathfrak{O}_K=$the ring of the algebraic integers in K.

Let $N\geq 3$ be a rational integer. Write

$$GL^+(2,K)=\left\{\begin{pmatrix} a & b \\ c & d\end{pmatrix}\in M_{2,2}(K)\,|\,ad-bc \text{ is a totally positive number.}\right\};$$

$$\Gamma(1)_K=\left\{\begin{pmatrix} a & b \\ c & d\end{pmatrix}\in M_{2,2}(\mathfrak{O}_K)\,|\,ad-bc \text{ is a totally positive unit of } \mathfrak{O}_K.\right\};$$

$$\Gamma(N)_K=\left\{\begin{pmatrix} a & b \\ c & d\end{pmatrix}\in \Gamma(1)_K\,|\,a-1\equiv d-1\equiv b\equiv c\equiv 0 \pmod{N\mathfrak{O}_K}.\right\};$$

$\mathfrak{H}^g=$the cartesian product of g copies of the complex upper half plane $\mathfrak{H}$. One sees that the group $\{\varepsilon\gamma\,|\,\gamma\in \mathrm{SL}(2,\mathfrak{O}_K).\ \varepsilon \text{ is a unit of } \mathfrak{O}_K.\}$ is a finite index subgroup of $\Gamma(1)_K$. We let $\mathrm{GL}^+(2,K)$ act on $\mathfrak{H}^g$ in the usual manner (see e.g. Shimizu [18]). One has the complex analytic quotient space $\Gamma(N)_K\backslash\mathfrak{H}^g$. Let φ be the Euler phi function. By the theory of moduli of abelian varieties with real multiplication by $\mathfrak{O}_K$ with level N structure (cf. Rapoport [17]) there exists a scheme $\mathcal{M}(N)$ of moduli over $\operatorname{Spec}\boldsymbol{Z}[1/N]$ such that $\mathcal{M}(N)\times_{\operatorname{Spec}\boldsymbol{Z}[1/N]}\operatorname{Spec}\boldsymbol{C}=\bigcup_{i=1}^{\varphi(N)}\Gamma(N)_K\backslash\mathfrak{H}^g$. This right side is a disjoint union of $\varphi(N)$ copies of $\Gamma(N)_K\backslash\mathfrak{H}^g$. These copies correspond to the N-th roots of unity bijectively. Write $G_2^+(\mathfrak{O}_K)=\mathrm{GL}^+(2,K)\cap M_{2,2}(\mathfrak{O}_K)$ which is a monoid. Write $HR(\Gamma(N)_K, G_2^+(\mathfrak{O}_K))=$the Hecke ring with respect to the pair $(\Gamma(N)_K, G_2^+(\mathfrak{O}_K))$. (For the definition of Hecke rings, see e.g. Shimura [19].) By the theory of Ash *et al.* [2] and Hirzebruch [14]