50. *The Differentiable Pinching Problem and the Diffeotopy Theorem*

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1. Introduction. Let (M, g) be a complete, simply connected riemannian manifold of dimension n. The purpose of this note is to announce the following theorem and to state the main idea of the proof. A detailed account will be published elsewhere [9].

Theorem 1. *Let (M, g) be a 0.681-pinched riemannian manifold. Then M is diffeomorphic to the standard sphere S^n.*

We say that (M, g) is a δ-pinched riemannian manifold, if the sectional curvature K satisfies $\delta \leqq K \leqq 1$.

A riemannian manifold with $\frac{1}{4} < K \leqq 1$ is homeomorphic to the standard sphere by the sphere theorem [1, 5]. The discovery of exotic spheres by Milnor gave rise to the question when the conclusion in the sphere theorem could be replaced by diffeomorphism. We call it the differentiable pinching problem. For the first time, Gromoll [2], Calabi and Shikata [7] gave some results on this problem. Later on, their results were improved as follows:

Theorem (Sugimoto-Shiohama [8]). *Let (M, g) be a 0.87-pinched riemannian manifold. Then M is diffeomorphic to the standard sphere.*

Theorem (Im Hof-Ruh [4]). *There exists a decreasing sequence δ_n with limit $\delta_n \to 0.68$ as n tends to infinity such that the following assertion holds:*

If (M, g) is a δ_n-pinched riemannian manifold of dimension n, and μ: $G \times M \to M$ is an isometric action of the Lie group G on M, then

(1) *there exists a diffeomorphism $F: M \to S^n$,*

(2) *there exists a homomorphism $\psi: G \to O(n+1)$ such that*

(3) *$\psi(g) = F \circ \mu(g, \cdot) \circ F^{-1}$ for all $g \in G$.*

We are interested in a pinching number independent of dimension of M. In Im Hof-Ruh's result, if we take the number δ independent of dimension of M, then δ becomes considerably large, i.e., $\delta = 0.98$ for $n > 5$. Our pinching number 0.681 is almost same as the number limit $\delta_n = 0.68$ given by Im Hof-Ruh. But their numbers are determined by quite different equations from each other.

2. Ideas. Sugimoto-Shiohama's beginning idea was as follows: A complete, simply connected δ-pinched riemannian manifold M^n is diffeomorphic to the standard sphere S^n if a diffeomorphism $f: S^{n-1} \to S^{n-1}$,

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