37. *On Certain Homotopy-homomorphic Elements of* $\pi_{n+1}(X)$

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§0. **Introduction.** Let X be a topological space with a base point x_0 and let $\Omega(X)$ be the loop space of X at x_0. We give $\Omega(X)$ the constant loop at x_0 as a base point. As well-known there exists the isomorphism: $\pi_{n+1}(X) \to \pi_n(\Omega X)$. We identify elements of these groups by this isomorphism. Now let a, b be given integers and $\mu: S^n \times S^n \to S^n$ be a map of type (a, b), i.e. such that $\mu(x, *)$ and $\mu(*, y)$ are maps $S^n \to S^n$ of degree a and b respectively. We call an element α of $\pi_{n+1}(X)$ a μ-homomorphic element (or to be μ-homomorphic) if and only if

$$\alpha(\mu(x, y)) = \omega(\alpha(m_a(x)), \alpha(m_b(y)))$$

where ω denotes the usual multiplication in $\Omega(X)$ and m_a is a map: $S^n \to S^n$ of degree a (in fact $m_a(x) = \mu(x, *)$).

In this note our purpose is to find an obstruction for determining to be μ-homomorphic. As a result we prove

Theorem 1. *For an element α of $\pi_{n+1}(X)$, α is μ-homomorphic if and only if $\alpha_*(c(\mu)) = 0$ where $c(\mu)$ denotes the Hopf construction as defined by James* ([2]).

An analogous problem has been considered in case of $\pi_3(G)$ for compact connected Lie groups G and $(a, b) = (1, 1)$ by Takahashi ([3]).

Our obstruction defines a correspondence

$$\chi: \pi_n(\Omega(X)) \longrightarrow \pi_{2n}(\Omega(X)).$$

This correspondence χ is not neccesarily homomorphic. We prove

Theorem 2. *χ is homomorphic if $\Omega(X)$ is a homotopy commutative Hopf space under the usual multiplication.*

§1. **An obstruction.** Denote with $\bar{\alpha}$ the adjoint element of $\alpha \in \pi_n(\Omega(X))$, and consider two maps: $S^n \times S^n \to \Omega(X) \times \Omega(X)$ in the following diagram:

$$(1) \qquad \begin{array}{ccccc} S^n \times S^n & \xrightarrow{\mu_1 \times \mu_2} & S^n \times S^n & \xrightarrow{\alpha \times \alpha} & \Omega(X) \times \Omega(X) \\ \mu \downarrow & & & & \downarrow \omega \\ S^n & & \longrightarrow & & \Omega(X) \end{array}$$

where $\mu_1(x) = \mu(x, *)$ and $\mu_2(y) = \mu(*, y)$. These two maps, $\alpha(\mu(x, y))$ and $\omega(\alpha(\mu_1(x)), \alpha(\mu_2(y)))$ coincides with each other on the sub-space $S^n \vee S^n$, so we have the difference element $\chi(\alpha) \in \pi_{2n}(\Omega(X))$ defined by these maps. Since $\Omega(X)$ is a Hopf space we have, from Puppe exact sequence,

Lemma 1. *α is μ-homomorphic if and only if $\chi(\alpha) = 0$.*

Thus it is sufficient for our purpose to describe $\chi(\alpha)$ as stated in