# 34. Construction of Certain Maximal p-ramified Extensions over Cyclotomic Fields

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§1. **Introduction.** Let $p$ and $m$ be, respectively, a fixed odd prime number and a fixed integer with $(p, m)=1$ and let $k=\boldsymbol{Q}(\cos(2\pi/m))$ and $K_\infty=k(\mu_{p^\infty})$. Denote by $\Omega_p$ the maximal pro-$p$ abelian extension over $K_\infty$ unramified outside $p$. Its odd part $\Omega_p^-$ contains the field

$$C=K_\infty(\varepsilon^{1/p^\infty} \mid \text{all circular units } \varepsilon \text{ of } K_\infty).$$

The extension $\Omega_p^-/C$ is of very delicate nature, and for example, when $k=\boldsymbol{Q}$, it is closely related to the Vandiver conjecture at $p$. We shall give a system of generators for the extension $\Omega_p^-/C$ (except for its "$\omega_p$-component") by using the theory of special units of F. Thaine [3].

§2. **Statement of the results.** Fix an *even* $\boldsymbol{Z}_p$-valued character $\chi$ of $\Delta_p=\mathrm{Gal}(k(\mu_p)/k)$, and let $\chi'$ be the *odd* character associated to $\chi$, i.e., $\chi'=\omega_p\cdot\chi^{-1}$. Here, $\omega_p$ is the Teichmüller character of $\Delta_p$. Since the Galois group $\Delta_p$ acts on the pro-$p$ abelian groups $\mathrm{Gal}(\Omega_p^-/K_\infty)$ and $\mathrm{Gal}(C/K_\infty)$ in the usual manner, we can decompose them by the $\Delta_p$-action. Let $\Omega_p(\chi')$ be the maximal intermediate field of $\Omega_p^-/K_\infty$ fixed by the $\psi$-components $\mathrm{Gal}(\Omega_p^-/K_\infty)(\psi)$ for all odd $\boldsymbol{Z}_p$-valued characters $\psi$ of $\Delta_p$ except $\chi'$. Define the intermediate field $C(\chi')$ of $C/K_\infty$ similarly.

To give a system of generators of the extension $\Omega_p(\chi')/C(\chi')$, we have to recall from [2] and introduce some notations. For a while, we fix a natural number $n$ and let $K_n=k(\mu_{p^{n+1}})$. For an abelian group A and an integer $N$, we abbreviate the quotient $A/NA$ as $A/N$. Let $M$ be any power of $p$. Regarding $(\boldsymbol{Z}/M)[\Delta_p]$ as a subring of $(\boldsymbol{Z}/M)[\mathrm{Gal}(K_n/\boldsymbol{Q})]$, we decompose $(\boldsymbol{Z}/M)[\mathrm{Gal}(K_n/\boldsymbol{Q})]$ by the $\Delta_p$-action. Denote its $\chi$-component by $\Lambda_{n,\chi,M}$. Let $E_n$ and $C_n$ be, respectively, the group of units and that of circular units of $K_n$. By a theorem on units in a Galois extension and that $[E_n : C_n]<\infty$, we see that there exists a Galois stable submodule $C_n'$ of $C_n$ such that $C_n'$ is cyclic over the group ring $\boldsymbol{Z}[\mathrm{Gal}(K_n/\boldsymbol{Q})]$ and $[E_n : C_n']<\infty$. In the following, assume that $\chi\neq$trivial ($\chi'\neq\omega_p$). Since $\chi\neq$trivial, the $\chi$-component $(C_n'/M)(\chi)$ is free and cyclic over $\Lambda_{n,\chi,M}$ for any $M$. Let $p^{\delta(n,\chi)}$ be the exponent of $(E_n/C_n')(p)(\chi)$, and we abbreviate $\Lambda_{n,\chi,p^{2\delta(n,\chi)}}$ as $\Lambda_{n,\chi}$. For an integer $i$, we denote by $\zeta_i$ a fixed primitive $i$-th root of unity. Let

$$\xi_n(1)=\prod_{i\mid mp^{n+1}}((1-\zeta_i)(1-\zeta_i^{-1}))^{a_i}$$

be a fixed generator of $(C_n'/p^{2\delta(n,\chi)})(\chi)$ over the group ring $\Lambda_{n,\chi}$, here $a_i$ is an element of $\Lambda_{n,\chi}$. For a prime number $l$ with $l\equiv 1 \pmod{mp^{n+1}}$, define an