32. *Regular Duo Near-rings*

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1. Introduction. In ring theory, it is well known that regular duo rings are characterized in terms of quasi-ideals (see [1, 3, 4]). The purpose of this note is to extend the above result to a class of regular duo near-rings. As to terminology and notation, we follow the usage in [2].

2. Preliminaries. Let N be a near-ring, which always means right one throughout this note.

If A, B and C are three non-empty subsets of N, then AB (ABC) denotes the set of all finite sums of the form $\sum a_k b_k$ with $a_k \in A$, $b_k \in B$ ($\sum a_k b_k c_k$ with $a_k \in A$, $b_k \in B$, $c_k \in C$).

A *right N-subgroup* (*left N-subgroup*) of N is a subgroup H of $(N, +)$ such that $HN \subseteq H$ ($NH \subseteq H$). A *quasi-ideal* of a zero-symmetric near-ring N is a subgroup Q of $(N, +)$ such that $QN \cap NQ \subseteq Q$. Right N-subgroups and left N-subgroups are quasi-ideals. The intersection of a family of quasi-ideals is again a quasi-ideal.

An element n of N is said to be *regular* if $n = nxn$ for some $x \in N$, and N is called *regular* if every element of N is regular.

Lemma 1. *Let N be a regular zero-symmetric near-ring. Then the following assertions hold:*

(i) *For every quasi-ideal Q of N, $Q = QNQ = QN \cap NQ$.*

(ii) *For every right N-subgroup R and left N-subgroup L of N, $RL = R \cap L$.*

Proof. (i) Let Q be a quasi-ideal of N, that is, $QN \cap NQ \subseteq Q$. By the regularity of N, $Q \subseteq QNQ$. Moreover we have $QNQ \subseteq QN$ and $QNQ \subseteq NQ$. Hence it follows that $Q \subseteq QNQ \subseteq QN \cap NQ \subseteq Q$. Thus $Q = QNQ = QN \cap NQ$.

(ii) Let R and L be right and left N-subgroups of N, respectively. Then $RL \subseteq R \cap L$ always holds. So we have to show only that an arbitrary element n of the intersection $R \cap L$ lies in RL. By the regularity of the element n, there exists an x in N such that $n = nxn$. Since $n \in R$ and $xn \in L$, we have $n = nxn \in RL$.

For an element n of a near-ring N, $(n)_r$ ($(n)_l$) denotes the principal right (left) N-subgroup of N generated by n, and $[n]$ denotes the subgroup of $(N, +)$ generated by n.

Lemma 2. *Let N be a near-ring with identity and n an element of N. Then $(n)_r = [n]N$ and $(n)_l = Nn$.*