31. A Two-Parameter Quantization of GL(n)

(*Summary*)

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1. One-parameter quantizations (or q-analogues) of the general linear group $GL(n)$ are known in two ways. The standard one arises as a dual Hopf algebra to the Drinfeld-Jimbo quantized enveloping algebra $U_q(\mathfrak{gl}(n))$ and is studied by many authors [5] [2] [6] [9] [8] [3]. The second one was defined by Dipper-Donkin [1]. One can define the quantum determinant for both quantizations. It is central in the first case, but not in the latter case.

We construct a two-parameter quantization $GL_{\alpha,\beta}(n)$ of $GL(n)$ depending on two units α, β in the base ring. The above known q-analogues are obtained as special cases by taking (q, q) and $(1, q)$ as (α, β) respectively. Further, we construct a two-parameter quantized enveloping algebra $U_{\alpha,\beta}$ associated with $GL_{\alpha,\beta}(n)$. The Drinfeld-Jimbo algebra $U_q(\mathfrak{gl}(n))$ is obtained as a quotient Hopf algebra of $U_{q,q}$.

2. We work over a commutative ring k. Let α and β be two units in k. Let $M_{\alpha,\beta}$ be the k-algebra defined by n^2 generators x_{ij} $(1 \leq i, j \leq n)$ and the following relations:

(2.1) $x_{ik}x_{ij} = \alpha x_{ij}x_{ik}$ if $j<k$.

(2.2) $x_{jk}x_{ik} = \beta x_{ik}x_{jk}$ if $i<j$.

(2.3) $x_{jk}x_{il} = \beta\alpha^{-1}x_{il}x_{jk}$, $x_{jl}x_{ik} - x_{ik}x_{jl} = (\beta - \alpha^{-1})x_{il}x_{jk}$ if $i<j$ and $k<l$.

The algebra $M_{\alpha,\beta}$ is a (non-commutative) polynomial algebra in x_{ij} in any ordering. This means if $w_1, \cdots, w_N$ $(N=n^2)$ is an arbitrary arrangement of x_{ij} $(1 \leq i, j \leq n)$, then the monomials $w_1^{e_1} \cdots w_N^{e_N}$ $(e_i \in \mathbf{N})$ form a free k-base for $M_{\alpha,\beta}$. If k is an integral domain, there is no non-zero divisor in $M_{\alpha,\beta}$.

The algebra $M_{\alpha,\beta}$ has a bialgebra structure such that

$$\Delta x_{ij} = \sum_{s=1}^{n} x_{is} \otimes x_{sj}, \quad \varepsilon x_{ij} = \delta_{ij}.$$

The *quantum determinant* $g = |X|$ is defined by

$$g = \sum_{\sigma} (-\beta)^{-l(\sigma)} x_{\sigma(1),1} \cdots x_{\sigma(n),n} = \sum_{\sigma} (-\alpha)^{-l(\sigma)} x_{1,\sigma(1)} \cdots x_{n,\sigma(n)}$$

where σ ranges over all permutations of n letters, and $l(\sigma)$ denotes the number of inversions. It is a group-like element, i.e., we have

$$\Delta g = g \otimes g, \quad \varepsilon g = 1.$$

It is a non-zero divisor of $M_{\alpha,\beta}$ and we have

$$x_{ij} g = (\beta\alpha^{-1})^{i-j} g x_{ij}.$$