20. Strong Continuity of the Solution to the Ljapunov Equation $XL-BX=C$ Relative to an Elliptic Operator L

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(Communicated by Kôsaku YOSIDA, M. J. A., March 13, 1989)

§1. Introduction. An operator equation, the so called Ljapunov equation, often appears in stabilization studies of linear parabolic systems. The equation is written as $XL-BX=C$, where the operators L, B, and C are given linear operators acting in separable Hilbert spaces, and are derived from a specific boundary feedback control system [6, 7, 8]. A general stabilization scheme for an unstable parabolic equation has been established in [6]. The parabolic equation containing L as a coefficient operator is often affected by small perturbations which may be sometimes interpreted as errors in mathematical formulation of a physical system. In such a case, does the feedback scheme still work for stabilization of the perturbed equation? A study of continuity of a solution X relative to L is fundamental to answer the question. It is the purpose of the paper to examine the continuity of X. We will see in §2 below an affirmative result on this problem.

Let us specify the operators L, B, and C. $\mathcal{L}$ will denote a strongly elliptic differential operator of order 2 in a connected bounded domain Ω of $\mathbb{R}^m$ with a finite number of smooth boundaries Γ of $(m-1)$-dimension;

$$\mathcal{L}u=-\sum_{i,j=1}^{m}\frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right)+\sum_{i=1}^{m}b_i(x)\frac{\partial u}{\partial x_i}+c(x)u,$$

where $a_{ij}(x)=a_{ji}(x)$, $1\leq i,\ j\leq m$, and for some positive δ

$$\sum_{i,j=1}^{m}a_{ij}(x)\xi_i\xi_j\geq\delta|\xi|^2,\quad \xi=(\xi_1,\cdots,\xi_m),\quad x\in\Omega.$$

Associated with $\mathcal{L}$ is a generalized Neumann boundary operator τ;

$$\tau u=\frac{\partial u}{\partial\nu}+\sigma(\xi)u,$$

where $\partial/\partial\nu=\sum_{i,j=1}^{m}a_{ij}(\xi)\nu_i(\xi)\partial/\partial x_j$, and $(\nu_1(\xi),\cdots,\nu_m(\xi))$ indicates the outward normal at $\xi\in\Gamma$. Then, L is defined in $L^2(\Omega)$ by

$$Lu=\mathcal{L}u,\qquad u\in\mathcal{D}(L)=\{u\in H^2(\Omega)\,;\,\tau u=0 \text{ on } \Gamma\}.$$

All norms hereafter will be either $L^2(\Omega)$- or $\mathcal{L}(L^2(\Omega))$-norm unless otherwise indicated. As is well known [2], the spectrum $\sigma(L)$ lies in the interior of a parabola $\{\lambda=\sigma+i\tau\,;\,\sigma=a\tau^2-b,\ \tau\in\mathbb{R}^1\}$, $a>0$. Second, the general structure of the operator B is specified in the following lemma:

Lemma 1.1 [6]. *Let A be a positive-definite self-adjoint operator in a separable Hilbert space H_0 with a compact resolvent. Let $\{\mu_i^2, \zeta_{ij}\,;\, i\geq 1, 1\leq j\leq n_i\ (<\infty)\}$ denote the eigenpairs of A (μ_i^2 are labelled according to increasing order, and ζ_{ij} normalized). Define H and B as*