96. Orbi-maps and 3-orbifolds

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1. Definitions. An n-orbifold is a topological space locally homeomorphic to (an open set in $\boldsymbol{R}^n$)/(a finite group action) and each point of it is provided with an isotropy data. By the symbol $|X|$, we shall mean the underlying space of the orbifold X.

For studying orbifolds, we need a map between orbifolds which respects their orbifold structures. An orbifold X is *good* if $|X|$ is homeomorphic to (a manifold $\tilde{X}$)/(a properly discontinuous action). In this paper, orbifolds which we deal with are good orbifolds. All orbifolds will be assumed to be good unless otherwise specified. If $\tilde{X}$ is simply connected, the quotient map $p: |\tilde{X}|\to|X|$ is called the *universal orbi-covering*.

Let X and Y be orbifolds. Let $p: |\tilde{X}|\to|X|$ and $q: |\tilde{Y}|\to|Y|$ be the universal orbi-coverings. We introduce an orbi-map between X and Y as follows; By an *orbi-map* $f: X\to Y$, we shall mean a continuous map $h: |X|\to |Y|$ with a fixed continuous map $\tilde{h}: \tilde{X}\to\tilde{Y}$ which satisfies the following conditions:

(01) $$h\circ p=q\circ\tilde{h}.$$

(02) For each $\sigma\in\mathrm{Aut}(\tilde{X}, p)$, there exists a $\tau\in\mathrm{Aut}(\tilde{Y}, q)$ such that $\tilde{h}\circ\sigma=\tau\circ\tilde{h}$.

(03) There exists a point $\tilde{x}\in\tilde{X}-p^{-1}(\Sigma X)$ such that $\tilde{h}(\tilde{x})\in\tilde{Y}-q^{-1}(\Sigma Y)$.

2. Constructions and modifications of orbi-maps.

2.1. Theorem. *Let M be a compact* 2- *or* 3-*orbifold and N an orientable* 3-*orbifold such that the total space of the universal orbi-covering of* $\mathrm{Int}(N)$ *is homeomorphic to* $\boldsymbol{R}^3$. *Suppose $\varphi: \pi_1(M)\to\pi_1(N)$ is a homomorphism such that for any local group G_x of M, $\varphi(G_x)\not\cong A_5$. Then, there exists an orbi-map $f: M\to N$ such that $f_*=\varphi$.*

2.2. Theorem (*Transversal modification of dimension* 3). *Suppose M and N are compact* 3-*orbifolds such that N is containing a properly embedded,* 2-*sided,* 2-*suborbifold F such that* $\mathrm{Ker}(\pi_1(F)\to\pi_1(N))=1$, $\pi_2(F)=0$, *and the total space of the universal orbi-covering of* $\mathrm{Int}(N-F)$ *is homeomorphic to* $\boldsymbol{R}^3$. *Suppose $f: M\to N$ is any orbi-map such that for any local group G, $f_*(G)\not\cong A_5$. Then there exists an orbi-map $g: M\to N$ such that*

(1) *g is C-equivalent to f,*

(2) *each component of $g^{-1}(F)$ is a properly embedded,* 2-*sided, incompressible* 2-*suborbifold in M, and*

(3) *for properly choosen product neighborhoods $F\times[-1,1]$ of $F=F\times 0$ in N and $g^{-1}(F)\times[-1,1]$ of $g^{-1}(F)=g^{-1}(F)\times 0$ in M, g maps each fiber $x\times$*