91. *Second Microlocal Singularities and Boundary Values of Holomorphic Functions*

By Yasunori OKADA and Nobuyuki TOSE
Department of Mathematics, Faculty of Sciences, University of Tokyo

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1. Introduction. The theory of the second microlocalization was initiated by M. Kashiwara [9]. He constructed the sheaf of 2-microfunctions and introduced the notion of second singular spectrum for microfunctions (cf. J.M. Bony [1], Kashiwara-Laurent [10]). On the other hand, J. Sjöstrand defined second analytic wave front sets for distributions with FBI transformation (cf. Esser-Laubin [7, 8]).

This paper aims at clarifying the relation between second microlocal singularities of hyperfunctions (or microfunctions) along linear involutive submanifolds of the cotangent bundle and their expressions as boundary values of holomorphic functions.

We introduce a special class of profiles and corresponding tuboids. Then we give a necessary and sufficient condition for a point to be outside of the second analytic wave front set of a hyperfunction. In the subsequent note [12], we will study, with this result, several functorial properties of second analytic wave front sets and will show the two notions of second microlocal singularities are equivalent.

2. Second analytic wave front sets. Let M be an open subset of $\boldsymbol{R}^n_x$ with a complex neighborhood X in $\boldsymbol{C}^n_z$. Then $(x;\xi\cdot dx)$ denotes a point of the cotangent bundle T^*M of M. We identify M with the zero section $T^*_M M$ of T^*M, and set $\dot{T}^*M=T^*M\backslash M$. We define an involutive submanifold V of $\dot{T}^*M$ by

$$V=\{(x;\xi)\in \dot{T}^*M;\ \xi_1=\cdots=\xi_d=0\}.$$

To simplify the notation, we put

$$x'=(x_1,\cdots,x_d),\qquad x''=(x_{d+1},\cdots,x_n),$$
$$\xi'=(\xi_1,\cdots,\xi_d),\qquad \xi''=(\xi_{d+1},\cdots,\xi_n),\quad etc.$$

We take coordinates of $T_V T^*M$ as $(x;\xi''\cdot dx'';x'^*\cdot\partial/\partial\xi')$ with $x'^*=(x_1^*,\cdots,x_d^*)\in\boldsymbol{R}^d$.

Definition 2.1. I) Let $u(x)$ be a hyperfunction with compact support. Then we define the FBI (Fourier-Bros-Iagolnitzer) transform and the second FBI transform along V of u by

$$Tu(z,\lambda):=\int u(x)\exp\left\{-\frac{\lambda}{2}(z-x)^2\right\}dx,$$
$$T^2_V u(z,\lambda,\mu):=\int u(x)\exp\left\{-\frac{\lambda\mu}{2}(z'-x')^2-\frac{\lambda}{2}(z''-x'')^2\right\}dx.$$