85. *A Characterization for Paracompactness*

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Introduction. Recently [5, 6] the authors introduced the notion of $B(P, \lambda)$-refinability and used this idea to obtain characterizations for paracompact, subparacompact, metacompact, θ-refinable, collectionwise normal, collectionwise subnormal and strong-collectionwise subnormal spaces. In this paper more general results are obtained in this class of $B(LF, \lambda)$-refinable spaces.

The properties P considered in this paper will be discrete (D) and locally finite (LF). The symbol λ will denote any countable ordinal.

Definition 1. A space X is *$B(P, \lambda)$-refinable* provided every open cover $\mathcal{U}$ of X has a refinement $\mathcal{E}=\cup\{\mathcal{E}_\beta : \beta<\lambda\}$ which satisfies i) $\{\cup\mathcal{E}_\beta : \beta<\lambda\}$ partitions X, ii) for every $\beta<\lambda$, $\mathcal{E}_\beta$ is a relatively P collection of closed subsets of the subspace $X-\cup\{\cup\mathcal{E}_\mu : \mu<\beta\}$, and iii) for every $\beta<\lambda$, $\cup\{\cup\mathcal{E}_\mu : \mu<\beta\}$ is a closed set.

The collection $\mathcal{E}$ is often called a *$B(P, \lambda)$-refinement* of $\mathcal{U}$. Expandable and θ-expandable spaces have been studied in [3, 4, 10, 11].

Definition 2. A space X is *strong-collectionwise subnormal* (*CWSN*) provided every discrete collection $\mathcal{D}$ of closed subsets X has a pairwise disjoint G_δ-expansion which is also a θ-expansion of $\mathcal{D}$.

In [6] the authors have obtained the following.

Theorem 1. *For any strong-CWSN space X, the following are equivalent.*

(a) *X is subparacompact.*

(b) *X is metacompact.*

(c) *X is θ-refinable.*

(d) *X is $B(D, \omega)$-refinable.*

The following has been shown in [4].

Lemma. (a) *Every paracompact space is expandable.*

(b) *A space X is countably paracompact iff X is countably expandable.*

Theorem 2. *A space X is paracompact iff X is $B(LF, \lambda)$-refinable and expandable.*

Proof. The necessity is clear. To prove the sufficiency, assume that X is expandable and $B(LF, \lambda)$-refinable. Let $\mathcal{U}$ be an open cover of X, and $\mathcal{E}=\cup\{\mathcal{E}_\gamma : \gamma<\lambda\}$ a $B(LF, \lambda)$-refinement of $\mathcal{U}$. We use induction to construct a family $\mathcal{V}^*=\{\mathcal{V}_\gamma : \gamma<\lambda\}$ of collections of subsets of X satisfying

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