

9. Configuration of Divisors and Reflexive Sheaves

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0. Let X be a connected complex manifold of dimension ≥ 2 and X^1 a reduced but *reducible* divisor of X . In this note, by using the Cech-stratification theoretical method in [7], we construct sheaves in the title from X^1 . A main property of such sheaves is:

(*) They have X^1 as their determinantal divisor (cf. § 1). We see that the two highly important bundles on the projective spaces, Horrocks-Mumford and null correlation bundles (cf. [4] and [6]) are constructed in the above manner. We also form some other interesting sheaves which seem to belong to new classes (cf. § 2). This note is a report of our recent works, cf. [8]. Details will appear elsewhere.

1. **Construction.** Set $X^2 = \bigcup_{i \neq j} (X_i^1 \cap X_j^1)$ and $\dot{X} = X - X^2$, where X_i^1 runs through all irreducible components of X^1 . Then our sheaf, denoted by \mathcal{E} , is obtained as the direct image $i_* \dot{\mathcal{E}}$ of a bundle $\dot{\mathcal{E}}$ over \dot{X} , with the injection $i : \dot{X} \rightarrow X$. In order to form $\dot{\mathcal{E}}$ we take (1) two open subsets N_0, N_1 of \dot{X} satisfying $N_0 \cup N_1 = \dot{X}$ and (2) a non singular matrix $H \in \text{GL}_r(\Gamma(N_0 \cap N_1, \mathcal{O}))$, where $r = \text{rank } \dot{\mathcal{E}}$ and $\mathcal{O} = \text{structure sheaf of } X$. Then the bundle $\dot{\mathcal{E}}$ is the one determined by H . We choose N_0, N_1 and H in such a manner that properties of X^1 reflect closely to them. More precisely we impose the following condition on N_0, \dots :

(1. 1) $N_0 = X - X^1$ and $N_1 = \bigsqcup_{i \in \Delta_m} N_{1,i}$, where $N_{1,i}$ is an open neighborhood of $\dot{X}_i^1 := X_i^1 - X^2$ in \dot{X} . (Here $\Delta_m = \{1, \dots, m\}$ with $m = \text{the number of the irreducible components of } X^1$.)

(1. 2) $H \in M_r(\Gamma(N_1, \mathcal{O}))$ and $(\det H)_0 = \dot{X}^1 (= \bigsqcup_{i \in \Delta_m} \dot{X}_i^1)$.

We see immediately that there are frames e^i of $\mathcal{E}|_{N_i}$, $i=0, 1$, satisfying $e^0 = e^1 H$ in $N_0 \cap N_1$. This implies that $e^0 \subset \Gamma(X, \mathcal{E})$ and that X^1 is the determinantal divisor of $\mathcal{E} : (\det e^0)|_{\dot{X}} = \dot{X}^1$.

2. **Examples.** Here we assume that $\dim X \geq 3$. Also we assume that (1) there is a line bundle \mathcal{L} over X and sections $s_i \in \Gamma(\mathcal{L})$ such that $X_i^1 = (s_i)_0$ and (2) for each $I = \{i_1, \dots, i_s\}$ satisfying $X_I := \bigcap_{i \in I} X_i^1 \neq \emptyset$, $\text{codim}_X X_I = s$ and X_I is smooth.

2.1. First we consider two types of matrices $H \in M_2(\Gamma(N_1, \mathcal{O}))$ as follows. (In (2. 1, 2) below, $i \in \Delta_m$.)

$$(2. 1) \quad H_{1N_1, i} = \begin{bmatrix} 1 & t_i \otimes s_i / \prod_{j \in \Delta_m} s_j \\ 0 & s_i / s_{i+1} \end{bmatrix} \quad \text{where } t_i \text{ is an element} \\ \text{of } \Gamma(\mathcal{L}^{\otimes (m-1)}),$$

and